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AN ANALYSIS OF COST VERSUS PERFORMANCE RELATIONSHIPS FOR PHASED ARRAY RADARS

L. A. Rondinelli

PREPARED FOR:

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87

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The RAND Corporation
1700 MAIN ST • SANTA MONICA • CALIFORNIA • 90406

PREFACE

This Memorandum is part of RAND's continuing effort to develop cost estimating relationships for new equipments. The objective is to provide the cost analyst with a technically sound framework for evaluating and predicting the cost of phased array radars. The Memorandum represents an expansion of RM-3729-ARPA, Array Radars: Performance and Cost Trade-offs (U), by J. D. Mallett, July 1963, Secret, with the major emphasis now on the sensitivity of radar cost to changes in radar performance.

The study should be of use to the Air Force Systems Command and other Headquarters USAF organizations concerned with the development and procurement of systems deploying phased array radars.

Dr. Rondinelli is a consultant to The RAND Corporation.

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SUMMARY

An analysis is presented that relates cost of phased array radars to specific radar performance requirements. Phased array performance parameters are derived separately for surveillance and tracking applications in terms of system operational requirements. A suitable equation for phased array hardware cost estimation is introduced. By the use of Lagrange multipliers, optimum design formulas and minimum cost equations are derived separately for surveillance and tracking applications, as well as for a combined simultaneous search and track capability.

Several phased array cost examples are presented using a representative range of cost coefficients resulting from a study of existing radar technology and the limited cost data available.

Lastly, the utility of array thinning as a cost saving technique is analyzed by comparing the cost of an optimum thinned tracking array design to the corresponding cost of an optimum unthinned design for equal performance. In the case of thinned arrays, the radar cost equation was modified to include a cost term proportional to the area of the thinned receive array. It is shown that when the area sensitive cost term is included, an optimum thinning ratio will exist for certain classes of cost coefficients.

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CONTENTS

I. INTRODUCTION

The ability to predict with reasonable accuracy the cost of future weapon systems as a function of their performance requirements has been, and will remain, a key factor in military planning decisions. In the case of ballistic missile defense, the rapidly changing technology available to both the offense and defense compounds the difficulty of obtaining useful cost estimating relationships. In particular, the transition in defense systems from conventional high power radars to the more sophisticated phased array radar systems necessitates the development of new radar cost estimating methods. This is particularly important since the cost of the phased array radars proposed in current ballistic missile defense studies is a significant fraction of total system cost.

The technology of electronically scanned phased array radars has advanced steadily in the past decade to the point where there is no longer a question of the practicability of fabricating reliable phased array systems. However, as one might expect, the ability to predict accurately the cost of phased array radar systems is limited by the lack of a sufficient number of phased array system development programs from which to draw experience. If sufficient cost data did exist to permit accurate costs to be assigned to identifiable phased array radar subassemblies (such is not the case at present), the cost analyst could readily estimate the cost of proposed phased array radar designs. However, it is still desirable for both the cost analyst and systems analyst to have a systematic method by which the cost of phased array radars could be related to radar performance.

It is only by such relationships (i.e., cost estimating relationships) that the cost analyst can extrapolate from the cost of presently proposed phased array radars to the estimated cost of similar radars proposed for future defense systems.

The purpose of this Memorandum is to develop useful relationships between cost and performance requirements for phased array radars for both the surveillance and tracking functions.

Accordingly, in Section III appropriate phased array performance parameters are derived separately for both surveillance and tracking applications in terms of system operational requirements.

In Section IV a suitable equation for phased array cost estimation is introduced. A procedure is presented that employs Lagrange multipliers to derive optimum (i.e., minimum) cost equations separately for surveillance and tracking applications, as well as for a combined simultaneous search and track capability, subject to the constraints imposed by the respective performance parameters Γ_s, Γ_t, and $\Gamma_s + \Gamma_t$.

In Section V several phased array cost examples are presented based on cost coefficients resulting from a study of existing radar technology in correlation with the limited cost data available. These results are analyzed to test the sensitivity of the cost of the phased array radar to variations in (1) the required system performance parameters and (2) the cost coefficients appearing in the radar cost equations.

In Section VI the utility of array thinning as a cost saving technique is analyzed by comparing the cost of an optimum thinned array design to the corresponding cost of an optimum unthinned design for equal performance.

II. FUNDAMENTALS OF PHASED ARRAYS

The concept of producing an electronically steered radar beam by means of an array of individually phase-controlled radiating elements is well known. However, it might be appropriate to review some fundamentals for the reader who is unfamiliar with radar technology. It is not the purpose of this section to attempt a complete summary of the various types of phased array configurations.* Instead, only those generic differences will be discussed that are felt to have the greatest influence on overall radar cost. In discussing beam forming techniques, it will often be convenient to refer to the array in terms of its acting as a transmitting antenna, with the understanding that similar statements will apply to a receiving array.

Perhaps the best way to bridge the gap in an introductory discussion of phased array radars is to begin by considering briefly their forerunner, the conventional mechanically scanned antenna. In radars employing mechanical scanning, the radar beam is steered by a physical rotation of the antenna structure. In applications requiring large antenna apertures and rapidly steerable beams, mechanical problems associated with the antenna drive mechanism become at best formidable and often insurmountable. Since many conventional antennas consist of a small primary aperture, which in turn illuminates a larger secondary reflecting aperture, it is possible to steer the beam by rotating only the smaller primary feed. Even this innovation

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*For an example of such a summary, see J. L. Allen, "Array Radars, A Survey of Their Potential and Their Limitations," The Microwave Journal, May 1962.

is inadequate, however, for applications requiring the steering of radar beams in times of the order of a millisecond or less.

The phased array radar circumvents the inertia limitation of mechanical scanning through the added degree of freedom in control of antenna aperture excitation provided by the use of multiple independently driven radiating elements. Scanning of the antenna beam is achieved by adjusting the relative phase of the signals exciting each successive element of the array in a predetermined manner. The time required to change the beam position is now determined by the speed with which the phases of the individual element excitations can be altered. This time is characteristically of the order of microseconds. In addition to increased scanning speed, phased arrays also have the advantage of being able to transmit successive beams in widely spaced directions, thereby permitting greater system design flexibility.

The simplest phased array configuration consists of a linear array of elements. Because of the linear array geometry it is possible to control only one of the two angles defining the beam position with such an array. The radiation pattern in the plane orthogonal to the linear array dimension is determined by the type of individual radiator employed. The pattern in this orthogonal plane is generally broad, and, more important, it cannot be electronically scanned.

To form a narrow pencil beam that can be scanned in two dimensions, the array geometry must be either planar or three-dimensional. An example of the relation between the radiated power pattern $P(\theta, \phi)$ and the element excitations I_{nm} for a planar array (shown in Fig. 1)

$$OA = md_y \sin\theta \sin\phi$$

$$OB = nd_x \sin\theta \cos\phi$$

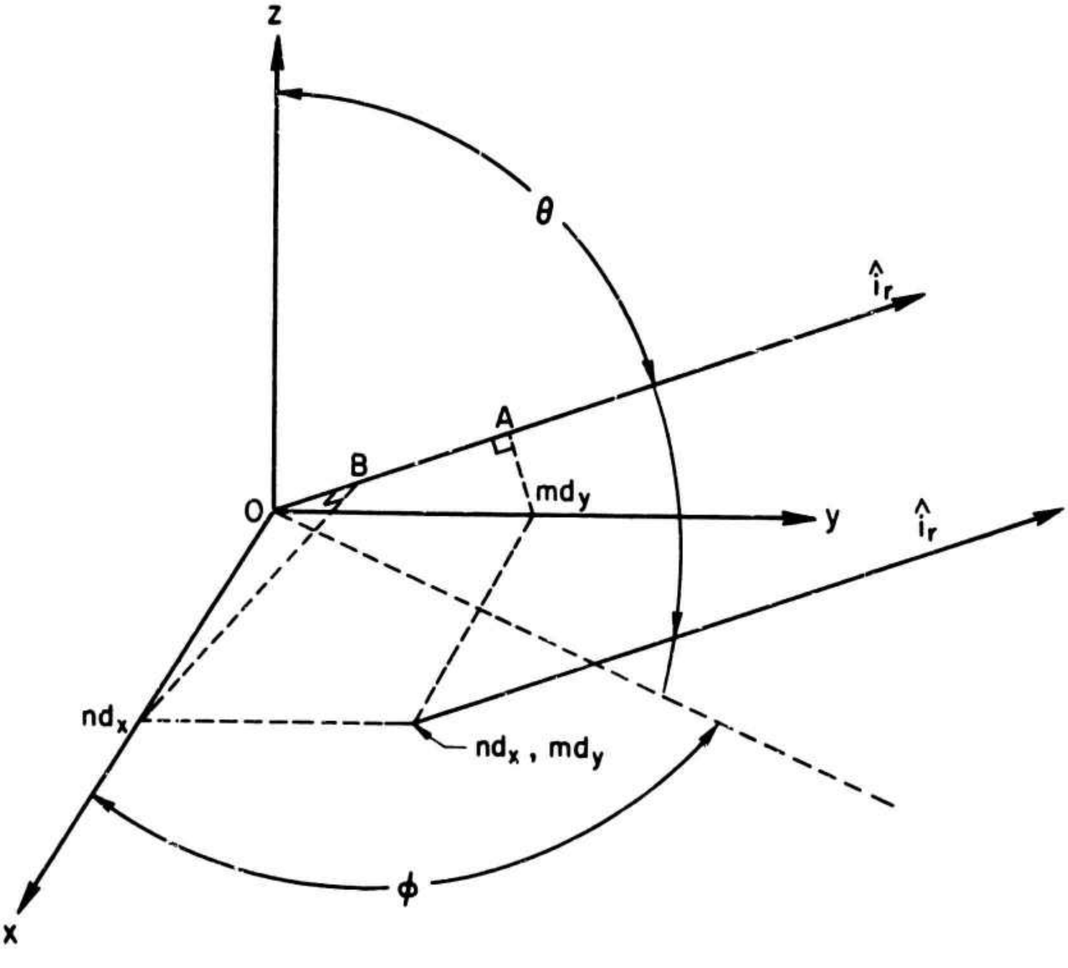

Fig. 1--Planar array geometry

is given in Eq. (1):

(1) $P(\theta, \phi)$

$$= \left|\sum_{n}^{N}\sum_{m}^{M}\right| I_{mn}\Big| e^{i(n\alpha+m\beta)} e^{ik[n(d_x \sin\theta\cos\phi)+m(d_y \sin\theta\sin\phi)]}\Big|^2 .$$

Beam scanning in a planar array is accomplished by varying the slope of a linear phase function along either or both of the orthogonal coordinate axes of the array. To point the beam in the coordinate direction (θ_0, ϕ_0), the differential phase shifts α and β must be chosen so that

$$\alpha = -kd_x \sin\theta_0 \cos\phi_0,$$

$$\beta = -kd_y \sin\theta_0 \sin\phi_0.$$

For purposes of a cost-oriented discussion, it is appropriate to distinguish the various array types on the basis of the amount of complex (and generally costly) electronic equipment associated with each element of the array. This is based on the intuitive assumption that for large arrays of many elements, it is the cost associated with each element that will have the greatest effect on overall radar cost. In order to appreciate the influence on phased array cost implied by various design alternatives, it might be advisable to list the various "components" that in the most general case, would be duplicated for each element of the array.

In general, a transmitter module* could include all of the

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*The antenna element and its associated components are usually referred to as a module.

following items:

(a) Antenna element.

(b) Phase shifter and associated control circuitry.

(c) Output amplifier.

(d) Cabling.

(e) Monitoring circuitry.

Likewise, a typical receiver module could include the following items:

(a) Antenna.

(b) Phase shifter and associated control circuitry.

(c) Radio-frequency (r-f) amplifier.

(d) Mixer.

(e) Intermediate-frequency (i-f) amplifier.

(f) Cabling.

(g) Monitoring circuitry.

In the case of a transmitter module, it is usually the phase shifter and the output amplifier that dominate total module cost. For a receiver module, the phase shifter, the r-f amplifier, and the mixer usually make the major contribution to module cost. Several techniques can be used to reduce the number of individual amplifiers and/or phase shifters (with respect to the number of array elements) while still retaining much of the flexibility of the more general array configuration. To reduce the number of individual amplifiers required, a corporate feed structure is most often used. Techniques for reducing the required number of phase shifters differ according to required signal bandwidth. Examples are

- Frequency scanning (narrow band scanning).
- Use of quasioptical beam forming or pure time delay insertion (wide band scanning).

These techniques will be discussed in greater detail below. For purposes of discussion it is convenient to describe these techniques in terms of their use in a linear array, although the same techniques are generally applicable to planar and three-dimensional arrays.

The module cost associated with the use of output power amplifiers can be significantly reduced by using a corporate feed structure (a series of power-dividing junctions with connecting transmission lines) through which all of the elements are excited from a single transmitter and driver stage. A major cost saving will result because, while vacuum tubes are used, it is usually less costly to generate average power in one high power transmitter than in many lower power units (for the same total average radiated power). This trend may be reversed when solid state transmitters become available.

In conjunction with the use of the corporate feed, each element of the array has a separate phase shifter. In this approach each phase shifter is required to handle (1/N)-th of the total peak and average power. In high power applications this will result in more costly phase shifters. As an alternative, it is possible to perform the required phase shifting at low power, either at i-f (followed by a mixer) or r-f, but then it is necessary to have an output power amplifier for each element.

For narrow band signal requirements, the simplest means of eliminating the need for individual element phase shifters and individual amplifiers is to employ frequency scanning. In this technique the elements are excited by means of a series of taps along a transmission line that is driven from one end. The elements of the array are generally spaced on the order of a half wavelength apart to prevent the formation of extraneous radiated beams. However, the length of transmission line between elements is made many wavelengths long, so that a small change in frequency results in a relatively large change in the incremental phase shift between successive elements and a corresponding scanning of the radiated beam. The fact that the beam pointing direction is frequency sensitive indicates that this technique is inherently narrow band in nature and therefore is restricted to applications in which wide band signals are not required. A further disadvantage of frequency scanning is that it precludes the use of bandwidth for other operational purposes such as pulse compression (i.e., improved range resolution) and frequency agility (i.e., jamming countermeasures).

A wide bandwidth technique that eliminates the need for individual element phase shifters is to use quasioptical techniques (i.e., microwave lenses) to develop desired phase fronts to excite pickup antennas, which in turn feed the radiating antenna array. The spherical Luneberg lens is an example of such a phase-computing lens for use with spherical arrays. Each of a series of appropriately positioned exciting horns (one for each beam position) on the input hemisphere will produce the required phase front at the pickup horns on the output hemisphere. The signal from each pickup horn drives a power amplifier, which in turn drives an element on the radiating spherical surface.

Precisely manufactured equal line length cables are used between the output elements of the computing lens and the transmitting elements. Since this technique makes use of pure time delay to develop the desired phase front, beam positioning is frequency independent; hence wide band signals can be accommodated.

It should be noted that there is generally a significant cost difference between narrow band and wide band scanning techniques. Wide bandwidth operation requires "time delay scanning" which may be achieved by quasioptical beam forming or by the insertion of real delay in each element. Both of these techniques tend to be considerably more costly than narrow band phase or frequency scanning techniques.

Even from this cursory discussion it is clear that there are a variety of "internal" design choices (each resulting in differences in system cost) from which to choose to satisfy any specified set of "external" performance requirements.

In the next two sections both the performance and cost formulas for phased array radars will be introduced in terms of externally measurable array parameters without regard to internal design features. The influence of internal design alternatives on the various cost coefficients in the general cost formula will be considered in Section V.

III. PERFORMANCE

SURVEILLANCE

The performance of surveillance radars may be measured by the single scan detection probability (i.e., the probability of detecting the return from a target in the presence of thermal noise during a single scan of the surveillance volume), although this is by no means the only measure of a surveillance radar's performance. Surveillance performance may also be specified, for example, in terms of the cumulative probability of detection attained on a target as a function of its range, but this measure is less general since it presupposes that the target is radially approaching the radar. For the purposes of this study, single scan detection probability appears to be a more satisfactory performance indicator. The single scan detection probability is related to the signal-to-noise ratio (i.e., the ratio of the energy received from the target to the average noise power density per cycle). This ratio, denoted as E/N_0, is given by

$$\frac{E}{N_0} = \frac{\overline{P}G_t A_e \sigma T_d}{(4\pi)^2 R^4 k T_{eff} L}, \tag{2}$$

where $\overline{P}$ = average transmitter power (in watts),

G_t = gain of transmitting antenna,

A_e = effective area of receiving antenna (in square feet),

σ = target cross section (in square meters),

T_{eff} = effective noise temperature of receiver referred to the antenna (in $^{\circ}$K),

T_d = dwell time, i.e., time that the radar illuminates the target during each scan (in seconds),

L = system losses,

k = Boltzmann's constant,

R = target range (in nautical miles).

An alternate expression for E/N_0, which is perhaps more appropriate when the surveillance requirement is stipulated in terms of the need to search a solid angle Ω once every T_f seconds, is

$$\frac{E}{N_0} = \frac{\overline{P}A_e\sigma}{4\pi R^4(kT_{eff}L)} \frac{T_f}{\Omega} . \tag{3}$$

This follows directly from Eq. (2) through substitution from the following relations:

$$G_t = \frac{4\pi}{\omega} = \frac{4\pi A_{et}}{\lambda^2} , \tag{4a}$$

$$T_d = T_f\frac{\omega}{\Omega} , \tag{4b}$$

where Eq. (4a) may be taken to be the definition of ω as the effective solid angle of the radiation pattern of the transmitting antenna. Equation (4a) also contains the defining relation between gain and effective antenna aperture.

From Eq. (3) we see the well-known result that during search the signal-to-noise ratio (and hence the detection probability) depends on the radar's power-aperture product ($\overline{P}A_e$) and not on the gain of the transmitting antenna.

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*This tacitly assumes that one or more receive beams is formed whose envelope matches the transmitter beam, and that both the transmit and receive beams are less than or equal to Ω.

In the case of phased array radars, the average power $\overline{P}$, the effective receiving aperature A_e, and the transmitter gain G_t may be replaced by the following expressions:

$$\overline{P} = \overline{p}_t N_t \ , \tag{5}$$

$$A_e \cong N_r \frac{\lambda^2}{4} \ , \tag{6}$$

$$G_t \cong N_t \pi \ , \tag{7}$$

where $\overline{p}_t$ = average transmitted power per element,

N_t = number of transmitter elements,

N_r = number of receiver elements.

Substituting Eqs. (5) and (6) into Eq. (3) and rearranging terms yields

$$\Gamma_s = \overline{p}_t N_t N_r = \frac{16\pi R^4 k T_{eff} L}{\lambda^2 \sigma} \cdot \left(\frac{\Omega}{T_f}\right) \cdot \left(\frac{E}{N_0}\right) \cong \frac{4}{\lambda^2} \overline{P} A_e \ . \tag{8}$$

The reason for the introduction of the search parameter $\Gamma_s = \overline{p}_t N_t N_r$ will become clear when the question of phased array cost is considered.

TRACKING

The tracking performance requirement may be formulated in terms of a specified accuracy in the measurement of the position of a target at a given range. The variance in total position measurement, at range R, is

$$\sigma_p^2 = R^2\left(\sigma_{\theta_x}^2 + \sigma_{\theta_y}^2\right) + \sigma_r^2 \ . \tag{9}$$

Manasse* has shown that the maximum accuracies (i.e., minimum standard deviations) of the angle and range measurements in the presence of thermal noise are given by

(10) $$\sigma_{\theta_x} = \frac{\sqrt{3}\theta_{0_x}}{\pi\sqrt{\frac{2E}{N_0}}} , \quad \sigma_{\theta_y} = \frac{\sqrt{3}\theta_{0_y}}{\pi\sqrt{\frac{2E}{N_0}}} , \quad \sigma_r = \frac{\sqrt{3}\Delta R}{\pi\sqrt{\frac{2E}{N_0}}} ,$$

where

(11) $$\theta_{0_x} = \frac{\lambda}{L_x} , \quad \theta_{0_y} = \frac{\lambda}{L_y} , \quad \Delta R = \frac{cT_{eff}}{2} .$$

Here L_x and L_y are the respective linear dimensions of the receive antenna along the orthogonal principal planes in which the beam widths θ_{0_x} and θ_{0_y} are measured; T_{eff} is the effective pulsewidth (i.e., approximately equal to the reciprocal of the instantaneous bandwidth); λ is the radar wavelength; and c is the velocity of light. Substituting Eq. (10) into Eq. (9), we have

(12) $$\sigma_p^2 = R^2\left[\sigma_{\theta_x}^2 + \sigma_{\theta_y}^2 + \left(\frac{\sigma_r}{R}\right)^2\right]$$

$$= R^2 \frac{3}{2\pi^2 \frac{E}{N_0}}\left[\left(\frac{\lambda}{L_x}\right)^2 + \left(\frac{\lambda}{L_y}\right)^2 + \left(\frac{\Delta R}{R}\right)^2\right] .$$

For most tracking cases the $[(\Delta R)/R]^2$ term is small in comparison to the contribution due to angle errors, and may therefore be neglected. That is, it is generally true that

*R. Manasse, Summary of Maximum Theoretical Accuracy of Radar Measurements, Technical Series Report No. 2, Mitre Corporation, Bedford, Mass., April 1960.

(13) $$\left(\frac{\Delta R}{R}\right)^2 << \left(\frac{\lambda}{L_x}\right)^2 + \left(\frac{\lambda}{L_y}\right)^2 .$$

Assuming that this inequality is satisfied, it follows from Eq. (12) that tracking performance can be specified either in terms of σ_p or σ_{θ_x} and σ_{θ_y} at a designated range R. For this discussion, σ_{θ_x} and σ_{θ_y} will be used since radar tracking performance is usually specified in terms of angle accuracies rather than position accuracies.

For the present we may limit consideration to rectangular phased arrays of uniformly spaced rows and colums. Let the normalized row and column spacings be

(14) $$D_x = \frac{d_x}{\lambda} , \quad D_y = \frac{d_y}{\lambda} .$$

If N_x and N_y are, respectively, the number of rows and columns in the receive array, then

(15) $$L_x = N_x \lambda D_x \quad \text{and} \quad L_y = N_y \lambda D_y ,$$

and

(16) $$N_r = N_x N_y$$

is the number of elements in the receive array.

From Eqs. (10) the signal-to-noise ratio may be solved for in terms of σ_{θ_x} and σ_{θ_y} yielding

(17) $$\frac{E}{N_0} = \frac{3}{2\pi^2} \frac{\theta_{0_x} \theta_{0_y}}{\sigma_{\theta_x} \sigma_{\theta_y}} .$$

From Eqs. (11), (14), and (15),

$$\theta_{0_y} = \frac{\lambda}{L_y} = \frac{1}{N_y D_y}$$

and

$$\theta_{0_x} = \frac{\lambda}{L_x} = \frac{1}{N_x D_x} \cdot$$

Hence

(18)
$$\frac{E}{N_0} = \frac{3}{2\pi^2} \frac{1}{\sigma_{\theta_x} \sigma_{\theta_y}} \frac{1}{N_r D_x D_y} \cdot$$

Equating this expression for required signal-to-noise ratio to obtain given angle accuracies with Eq. (2) for the signal-to-noise ratio achievable on a single target at range R during a dwell time T_d yields (using Eqs. (5), (6), and (7)):

(19)
$$\Gamma_t = \bar{p}_t N_t^2 N_r^2 = \frac{96}{\pi D_x D_y} \frac{R^4}{\sigma\lambda^2 T_d} \frac{kT_{eff}L}{\sigma_{\theta_x} \sigma_{\theta_y}} ,$$

where Γ_t is the tracking parameter that relates the radar's performance to the measurable array parameters (i.e., $\bar{p}_t$, N_t, and N_r). In particular, for half-wavelength element spacings, $D_x = D_y \approx \frac{1}{2}$ so that

(20)
$$\Gamma_t = \bar{p}_t N_t^2 N_r^2 = \frac{384 R^4 kT_{eff} L}{\pi\sigma\lambda^2 \sigma_{\theta_x} \sigma_{\theta_y} T_d} \cdot$$

The specific requirements for σ_{θ_x} and σ_{θ_y} depend on the intended application of the tracking radar, and this need not be of concern for the present discussion.

At this point, a word of explanation might be in order to clarify the significance of the search and track parameters given in Eqs. (8)

and (20), respectively. The relevance of these equations is that they link the external radar performance parameters (i.e., R, σ, λ, σ_{θ_x}, etc.) to the measurable array parameters $\bar{p}_t$, N_t, and N_r, which in turn can be related to radar cost. The radar cost formulation will be presented in the following section, where radar cost will be derived as a function of $\bar{p}_t$, N_t, N_r, and either Γ_s or Γ_t or both depending on the operational function of the radar. Since Eq. (8) (for Γ_s) and Eq. (20) (for Γ_t) can then be used in conjunction with the cost formulas to scale cost versus performance, it is important to note that the range dependence in Eqs. (8) and (20) will be modified from R^4 to R^3 by the fact that the search frame time T_f (Eq. (8)) and the tracking dwell time T_d (Eq. (20)) will generally be linearly proportional to maximum range, assuming a fixed target velocity. This point will be explained further in the cost examples given in Section V.

IV. RADAR COST

FORMULATION

In attempting to formulate an expression for phased array radar cost, it is desirable to arrive at a formula that is sufficiently general so that it may be used irrespective of the following:

(a) The phase shifting technique used for beam scanning.

(b) The transmitter power distribution configuration.

(c) The physical geometry of the transmitting and receiving antenna arrays.

In this way, various design alternatives such as the following can be subsumed under a single cost formula:

(a) Using frequency scanning versus phase scanning versus pure time delay scanning.

(b) Using a single high power transmitter and a corporate feed network versus using as many lower power modules as there are elements (or some ratio in between).

(c) Combining the transmit and receive functions in a single array using duplexers versus using physically separate transmit and receive arrays.

However, since the radar cost will in general depend on the above design choices, it will be necessary (to the extent that our knowledge permits) to reflect this dependence by appropriate changes in the coefficients appearing in the general cost formula. Although this approach may appear to beg the issue, it has the distinct advantage of allowing an analysis of radar cost optimiza ion that is uniformly applicable to a wide variety of phased array system designs.

With this in mind, it was decided that the smallest possible set of phased array physical attributes that should be included in a general cost formula consists of the following:

(1) Average transmitted power, $\overline{P}_t$.

(2) Number of transmitter elements, N_t.

(3) Number of receiver elements, N_r.

Up to this point we have talked about the cost of phased array radars in a general way. Before proceeding further, some clarification is in order. In discussing costs of phased array radars, there are several cost categories that should be considered:

- Design and development cost.
- Hardware cost.
- Signal processing cost.
- Installation, checkout, and monitoring cost.
- Facilities and/or supporting structure cost.
- Maintenance and operating cost, program management.

Included in hardware cost will be the total cost to manufacture all components and assemble and factory test all subsystems of radar hardware. This will include the cost of any special purpose beam steering computers but not the cost of a general purpose signal processing computer that would be included under signal processing cost.

In applying these categories to phased array radars, the hardware cost can be easily related to the number of elements used in the array. Design and development cost is closely geared to the number of state-of-the-art components that the system will use; it is essentially independent of the <u>size</u> of the resulting array. Signal processing cost will depend more on the mission functions to be performed than on the size of the array radar. While maintenance and operating costs depend on the number of elements in the array, they are recurring costs and are best considered separately from the other categories, which represent investment cost. The cost of the building that houses the array and its ancillary equipment can be related to the area of the array (or faces) and to the hardness requirement. This is important only in the case of array thinning and where the hardening cost is significant. If the array area is small, the ancillary equipment operational area will require a separate building or an underground facility.

In attempting to relate radar performance to cost, we will be concerned primarily in this study with the hardware fabrication and installation cost.* In this way the radar cost will be proportional to the number of transmitter and receiver elements in the array, and the module costs will be less sensitive to the number of elements than would be the case if we prorated the cost of design and development, installation, checkout, monitoring, and facilities development among the total number of elements. This being the case, the following cost formula seems reasonable:

$$C = C_t N_t + C_p \bar{p}_t N_t + C_r N_r \ , \tag{21}$$

where C = hardware cost of a phased array radar system (in \$),

C_t = cost of manufacturing and assembling a transmitter module (in \$/element),

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*The nonhardware cost to deploy a phased array radar will be the subject of another study.

C_p = cost of producing average power (in \$/watt),

C_r = cost of manufacturing and assembling a receiver module (in \$/element),

and N_t, N_r, and $\bar{p}_t$ are as previously defined.

COST OPTIMIZATION

The approach to be used assumes that in designing an array radar one seeks to select those transmit and receive array configurations that achieve the desired performance at minimum cost. If the radar is to be used only for surveillance, then, as shown in the first part of Section III, the influence of the performance specifications is reflected through the required search parameter $\Gamma_s = \bar{p}_t N_t N_r$, which in the case of a filled array is equivalent to the power-aperture product PA_e divided by $\lambda^2/4$. If the radar is to be used only for tracking, then performance will be influenced by the gain of the transmitting antenna, and, as shown in the second part of Section III, the parameter of importance is $\Gamma_t = \bar{p}_t N_t^2 N_r^2$. For combined applications, where the same array radar is to be used for both search and track functions, it would be fortuitous if the minimum cost arrays for the search function were identical to the minimum cost arrays for tracking. In such combined applications, a minimum cost array configuration can be designed subject to the combined constraint imposed by both Γ_s and Γ_t. In the following three subsections, optimum array configurations will be derived separately for a search radar, a tracking radar, and a radar combining the search and track functions.

Optimum Search Array Radar

To derive the minimum cost array configuration, the method of Lagrange multipliers will be applied to Eq. (21), subject to the search performance constraint of Eq. (8).* The cost will be minimized by choosing the proper values for N_t, N_r, assuming that the cost coefficients C_t, C_r, C_p are specified and that $\bar{p}_t$ takes on one of several possible values established by available power tubes in a given frequency band. Obviously, C_t and C_r will also depend on the frequency band chosen.

Thus

(22) $$C = (C_t + C_p\bar{p}_t)N_t + C_rN_r + \xi(\bar{p}_tN_tN_r - \Gamma_s) ,$$

(23) $$\frac{\partial C}{\partial N_t} = (C_t + C_p\bar{p}_t) + \xi\bar{p}_tN_r = 0 ,$$

(24) $$\frac{\partial C}{\partial N_r} = C_r + \xi\bar{p}_tN_t = 0 ,$$

(25) $$\frac{\partial C}{\partial \xi} = \bar{p}_tN_tN_r - \Gamma_s = 0 .$$

Solving (23) for ξ and substitution into (24) yields

(26) $$N_r = \frac{(C_t + C_p\bar{p}_t)}{C_r} N_t .$$

Combining (25) and (26) yields, for the optimum number of transmitter and receiver elements,

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*L. Cartledge, "Minimum-cost Array Configurations," Appendix A, Part 3, Chapter IV, in Phased Array Radar Studies, Technical Report No. 299, M.I.T., Lincoln Laboratory, Lexington, Mass., 20 February 1963.

(27a) $$N_{t_{opt}} = \left(\frac{\Gamma_s}{\bar{p}_t} \cdot \frac{C_r}{(C_t+C_p\bar{p}_t)}\right)^{\frac{1}{2}},$$

(27b) $$N_{r_{opt}} = \left(\frac{\Gamma_s}{\bar{p}_t} \cdot \frac{(C_t+C_p\bar{p}_t)}{C_r}\right)^{\frac{1}{2}}.$$

Note that by substituting Eq. (26) into the cost formula of Eq. (21), we have

(28) $$C_{\substack{opt\\search}} = 2(C_t + C_p\bar{p}_t)\, N_{t_{opt}} = 2C_r N_{r_{opt}} .$$

Hence the optimum cost configuration occurs when the receive array costs as much as the combined cost of the transmit array and the cost of producing the requisite average power.

Substituting the expression for N_t from Eq. (27a) into Eq. (28) yields

(29) $$C_{\substack{opt\\search}} = 2\left[C_r(C_t + C_p\bar{p}_t)\frac{\Gamma_s}{\bar{p}_t}\right]^{\frac{1}{2}} .$$

And, from Eq. (8) we have

(30) $$\Gamma_s^{\frac{1}{2}} = \left(16\pi kT_{eff}L\,\frac{E}{N_0}\right)^{\frac{1}{2}} \frac{R^2}{\lambda\sigma^{\frac{1}{2}}}\left(\frac{\Omega}{T_f}\right)^{\frac{1}{2}} .$$

Since the optimum search cost is proportional to $\Gamma_s^{\frac{1}{2}}$ (for fixed cost coefficients and $\bar{p}_t$), we see that for a given required performance (i.e., fixed E/N_0) the cost of an optimum search array is proportional to

$$R^2 \left(\frac{\Omega}{\sigma T_f}\right)^{\frac{1}{2}} .$$

The dependence of cost on wavelength was deliberately omitted since the cost coefficients and $\bar{p}_t$ will probably be functions of λ.

<u>Optimum Tracking Array Radar</u>

Once again the method of Lagrange multipliers is applied to the cost formula of Eq. (21); however, in this case the constraint will be imposed by the tracking parameter Γ_t of Eq. (20).

Thus

(31) $$C = (C_t + C_p\bar{p}_t)N_t + C_rN_r + \xi(\bar{p}_tN_t^2N_r^2 - \Gamma_t) ,$$

(32) $$\frac{\partial C}{\partial N_t} = (C_t + C_p\bar{p}_t) + 2\xi\bar{p}_tN_tN_r^2 = 0 ,$$

(33) $$\frac{\partial C}{\partial N_r} = C_r + 2\xi\bar{p}_tN_t^2N_r = 0 ,$$

(34) $$\frac{\partial C}{\partial \xi} = \bar{p}_tN_t^2N_r^2 - \Gamma_t = 0 .$$

Solving Eq. (32) for ξ and substitution into Eq. (33) yields

(35) $$N_r = \frac{C_t + C_p\bar{p}_t}{C_r} N_t .$$

This is identical to Eq. (26) so that <u>once again for optimum design the total cost is equally divided between the transmit and receive functions</u>.

Combining Eqs. (34) and (35) yields for the optimum number of transmitter and receiver elements

(36a) $$N_{t_{opt}} = \left[\frac{\Gamma_t}{\bar{p}_t} \frac{C_r^2}{(C_t + C_p\bar{p}_t)^2}\right]^{\frac{1}{2}},$$

(36b) $$N_{r_{opt}} = \left[\frac{\Gamma_t}{\bar{p}_t}\left(\frac{C_t + C_p\bar{p}_t}{C_r}\right)^2\right]^{\frac{1}{2}}.$$

Again for the optimum cost array $N_{r_{opt}}$ is proportional to $N_{t_{opt}}$, and the total array cost is also proportional to $N_{t_{opt}}$. Substituting Eq. (36) into the cost formula yields

(37) $$C_{\substack{opt\\track}} = 2(C_t + C_p\bar{p}_t)N_{t_{opt}} = 2\left(\frac{\Gamma_t}{\bar{p}_t}\right)^{\frac{1}{2}}\left[C_r(C_t + C_p\bar{p}_t)\right]^{\frac{1}{2}}.$$

From Eq. (37) we see that the cost of the optimum tracking array is proportional to $\Gamma_t^{\frac{1}{2}}$. Hence, from Eq. (20),

(38) $$C_{\substack{opt\\track}} \propto \Gamma_t^{\frac{1}{2}} \alpha \left(\frac{1}{\sigma\sigma_{\theta_x}\sigma_{\theta_y}T_d}\right)^{\frac{1}{2}} R \quad \text{for single target.}$$

If instead of tracking one target in a dwell time T_d, as is assumed above, the requirement is to track M targets in the same total time, then Γ_t would be multiplied by M if the targets were all of the same cross section. In the more general case of M different targets at various ranges and having different required angular accuracies for each, we would have

(39) $$C_{\substack{opt\\track}} \propto \sum_{k=1}^{M} R_k\left(\frac{1}{\sigma_k\sigma_{\theta_{x_k}}\sigma_{\theta_{y_k}}T_{d_k}}\right)^{\frac{1}{2}}.$$

Once again, with respect to wavelength, all that can be said is that if the cost coefficients and power per module are the same at two separate frequencies, then from Eqs. (20) and (37) the cost of the optimum tracking array radar will vary as $1/\lambda^{\frac{1}{2}}$.

Optimum Combined Search and Track Array Radar

If the same array is to perform simultaneously the search and track functions, then it is possible to optimize the cost subject to the constraints imposed by both Γ_t and Γ_s.

If $\bar{p}_s$ and $\bar{p}_t$ denote the respective average powers per transmitter module required to perform the search and track functions, a suitable approach to cost optimization is to use the Lagrange multiplier approach with a constraint on the sum $\Gamma_s + \Gamma_t$. From Eqs. (8), (20), and (21), we have

$$C = C_t N_t + C_p(\bar{p}_t + \bar{p}_s)N_t + C_r N_r + \xi[\bar{p}_s N_t N_r + \bar{p}_t N_t^2 N_r^2 - (\Gamma_s + \Gamma_t)] \; . \tag{40}$$

Taking partial derivatives with respect to N_t, N_r, and ξ,

$$\frac{\partial C}{\partial N_t} = C_t + C_p(\bar{p}_t + \bar{p}_s) + \xi(\bar{p}_s N_r + 2\bar{p}_t N_t N_r^2) = 0 \; , \tag{41a}$$

$$\frac{\partial C}{\partial N_r} = C_r + \xi(\bar{p}_s N_t + 2\bar{p}_t N_t^2 N_r) = 0 \; , \tag{41b}$$

$$\frac{\partial C}{\partial \xi} = \bar{p}_s N_t N_r + \bar{p}_t N_t^2 N_r^2 - (\Gamma_s + \Gamma_t) = 0 \; . \tag{41c}$$

Solving Eq. (41b) for ξ, substituting the result into Eq. (41a), and simplifying the resulting expression by factoring yields:

$$(42) \qquad [\bar{p}_s + 2\bar{p}_t N_t N_r][N_t(C_t + C_p(\bar{p}_t + \bar{p}_s)) - N_r C_r] = 0 \, .$$

Since the first factor is always positive, the second bracketed factor must be zero to satisfy Eq. (42). This yields the following relationship between N_r and N_t:

$$(43) \qquad N_r = \frac{[C_t + C_p(\bar{p}_t + \bar{p}_s)]}{C_r} N_t \, .$$

This agrees in form with both Eqs. (26) and (35) if in each of these equations we replace the former symbol for average power per module, $\bar{p}_t$, by the sum $\bar{p}_t + \bar{p}_s$, which is the total average power per module in the combined case.

Substituting Eq. (43) into Eq. (41c) results in the following quartic equation in N_t:

$$(44) \qquad N_t^4 + \frac{\bar{p}_s}{\bar{p}_t} \frac{C_r}{[C_t + C_p(\bar{p}_t + \bar{p}_s)]} N_t^2 - \frac{(\Gamma_s + \Gamma_t)}{\bar{p}_t} \frac{C_r^2}{[C_t + C_p(\bar{p}_t + \bar{p}_s)]^2} = 0 \, .$$

It may be [illegible] at Eq. (44) has a single positive solution for N_t (a double root) given by

$$(45a) \qquad N_{t_{opt}} = \left\{ \frac{C_r}{2[C_t + C_p(\bar{p}_t + \bar{p}_s)]} \left[\left\{ \left(\frac{\bar{p}_s}{\bar{p}_t} \right)^2 + \frac{4(\Gamma_s + \Gamma_t)}{\bar{p}_t} \right\}^{\frac{1}{2}} - \frac{\bar{p}_s}{\bar{p}_t} \right] \right\}^{\frac{1}{2}} \, .$$

Combining (45a) with (43) yields for the optimum number of receiver modules

$$(45b)\quad N_{r_{opt}} = \left\{\frac{[C_t + C_p(\bar{p}_t + \bar{p}_s)]}{2C_r}\left[\left\{\left(\frac{\bar{p}_s}{\bar{p}_t}\right)^2 + \frac{4(\Gamma_s + \Gamma_t)}{\bar{p}_t}\right\}^{\frac{1}{2}} - \frac{\bar{p}_s}{\bar{p}_t}\right]\right\}^{\frac{1}{2}} .$$

Substituting these values into the cost equation yields

$$(46)\quad C_{\substack{opt\\combined}} = \left\{2C_r[C_t + C_p(\bar{p}_t + \bar{p}_s)]\left[\left\{\left(\frac{\bar{p}_s}{\bar{p}_t}\right)^2 + 4\left(\frac{\Gamma_s + \Gamma_t}{\bar{p}_t}\right)\right\}^{\frac{1}{2}} - \frac{\bar{p}_s}{\bar{p}_t}\right]\right\}^{\frac{1}{2}} .$$

The optimum cost formula in the combined case may easily be shown to agree with the separate cost optimization formulas for tracking (Eq. (37)) and search (Eq. (29)). For a tracking radar, if we set $\Gamma_s = 0$ and $\bar{p}_s = 0$ in Eq. (46), the result is identical to Eq. (37). In the case of a search radar, $\Gamma_t = 0$ and $\bar{p}_t = 0$. In this case we revert to Eq. (44), which becomes a quadratic in N_t. Solving for N_t yields an expression identical to Eq. (28), resulting in an optimum cost formula in agreement with Eq. (29).

Comparing Eqs. (45) and (46) for the optimum design combining search and track to the corresponding equations for a search radar* or a tracking radar,** it is clearly easier to scale cost versus performance in either of the separate cases than it is in the case of combining search and track. It was therefore decided that for simplicity, the cost examples discussed in Section V would be limited to separate search and track radars. In practice, in many cases, one or the other of the requirements (Γ_t or Γ_s) will dominate the design

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*Equations (27) and (29).

**Equations (36) and (37).

configuration and resulting cost. In such cases, the secondary function can be added for the small incremental cost required to produce the additional required average power. The resulting design, although nonoptimum, will differ only slightly from an optimum design. This point will be discussed further in Section V.

In summary, equations have been derived for optimum hardware cost separately for a search radar (Eq. (29)), a tracking radar (Eq. (37)), and a combined simultaneous search and track radar in terms of

- The cost coefficients C_t, C_r, C_p.
- The average power per element, $\bar{p}_t$.
- The performance parameters Γ_s (Eq. (8)) and Γ_t (Eq. (20)).

From a radar's operational performance specifications the parameters Γ_s and/or Γ_t may be directly evaluated from Eq. (8) and/or Eq. (20), respectively. To evaluate hardware cost, all that remains is to specify $\bar{p}_t$ and make appropriate estimates of the cost coefficients C_t, C_r, and C_p based on the limited cost data presently available. In Section V, a cost sensitivity analysis will be presented to exhibit the dependence of the hardware cost estimate on changes in performance requirements and uncertainties in the several cost coefficients. Finally, in Section VI, the efficacy of array thinning as a cost reduction technique will be investigated.

V. COST EXAMPLES

In this section several examples will be presented to illustrate the cost optimization design procedure outlined in Section IV. Separate array designs will be presented for both the search and track functions in terms of the respective operational requirements. Since the optimum design procedure presupposes knowledge of the module cost coefficients, and since in fact these costs may be little more than assumed values subject to large uncertainties prior to manufacturing, the resulting designs may be ex post facto nonoptimum. Accordingly, several examples of nonoptimum design configurations and cost estimates will be presented to determine the sensitivity of hardware cost to the lack of a priori knowledge of the module cost coefficients. Similarly, the sensitivity of optimum hardware cost to variations in the cost coefficients C_t, C_r, and C_p will be examined to determine whether or not any of the coefficients have a sufficiently more critical influence on total cost to warrant a more concerted effort in arriving at its estimate than is required for the other coefficients.

LONG-RANGE SURVEILLANCE RADAR

As an example of a search radar, consider a long-range surveillance radar designed to detect targets that penetrate a fan-shaped volume between some specified minimum and maximum ranges.

It will be assumed that targets of interest penetrate the search volume in the elevation direction with some constant transverse velocity V_t. If the coverage volume is $\Delta\phi$ in azimuth by $\Delta\theta$ deg in elevation, then the time for a target to traverse the coverage volume would be

(47) $$\Delta t = \frac{R \cdot \Delta\theta}{V_t} .$$

In a mechanically scanned radar the search routine must be tailored to the minimum traversal time corresponding to a target entering the surveillance volume at minimum range. This problem is alleviated with phased array radars due to the flexibility of beam steering available to the system designer. From Eq. (3) the average power will depend on the maximum target detection range. To reduce the required average power, and thereby reduce the radar cost, the frame time in Eq. (3) can be equated to the traversal time Δt of Eq. (47) for the maximum range of interest. In this way, the average power will only increase as R^3 instead of R^4. To ensure the detection of targets traversing the surveillance volume at minimum range, a low power subroutine can be included in the search program using either beam broadening or reduced frame time. These techniques will not significantly increase the radar cost and hence need not be considered further for our purposes.

If T_f is equated to Δt of Eq. (47) and substituted into Eq. (3), we have

(48) $$\overline{P} \cdot A_e \cdot \sigma = 4\pi R^3 k T_{eff} L \frac{\Omega V_t}{\Delta\theta} \left(\frac{E}{N_0}\right) .$$

Referring to the geometry of Fig. 2, we have

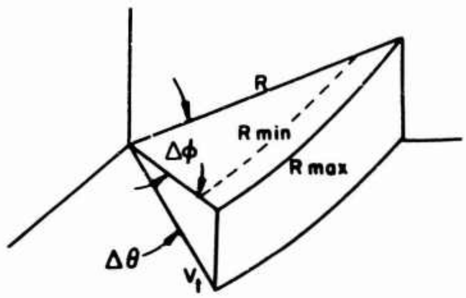


Fig. 2--Surveillance geometry

$$d\Omega = \frac{|dA|}{R^2} = \frac{R^2 \sin\theta \, d\theta \, d\phi}{R^2} ,$$

$$\Omega = \int_{\Omega_1}^{\Omega_2} \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta \, d\phi$$

$$= \Delta\phi \, (\cos\theta_1 - \cos\theta_2) ,$$

$$\Omega \simeq \Delta\phi \, \Delta\theta ,$$

or

(49) $$\frac{\Omega}{\Delta\theta} \simeq \Delta\phi .$$

As was indicated in Section III, the probability of detecting a target during a single scan, which we denote as P_D, is related to both the signal-to-noise ratio, E/N_0, and the false alarm probability, P_F. The exact form of the relationship between these parameters depends on the statistical description of the corrupting noise present at the receiver and the type of signal processing performed, and the type of target, i.e., fluctuating versus nonfluctuating cross section. Much research has been devoted to deriving various optimum detection schemes (e.g., sequential detection), based on assumedly available a priori statistical knowledge. Rather than dwell on such detailed questions, it will suffice for our purposes to use as representative performance the optimum detection curves derived by Manasse* for Gaussian noise and a nonfluctuating target model, as shown in Fig. 3. The curves of Fig. 3 can be used in conjunction with Eq. (48) to derive

*R. Manasse, The Application of the Theory of Signal Detectibility to Signals with Unknown Polarization and Phase, Group Report 32-25, M.I.T., Lincoln Laboratory, Lexington, Mass., August 1956.

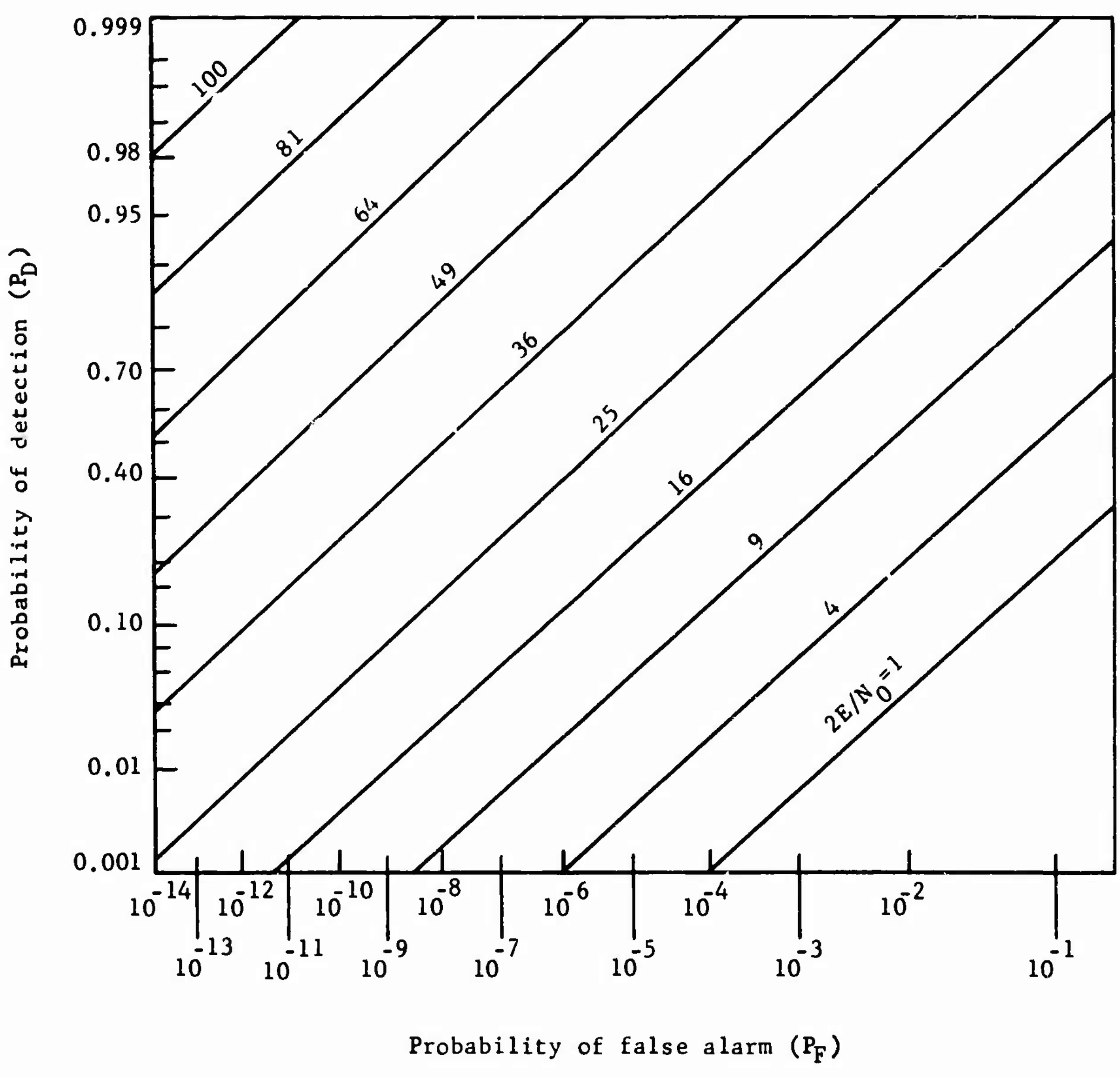


Fig. 3--Typical probability of detection (P_D) versus probability of false alarm (P_F) curves for optimum receiver (from Manasse) (known polarization, unknown phase)

curves of detection probability, P_D, versus range, R, for several values of the product $\overline{P} \cdot A_e \cdot \sigma$ and false alarm probabilities of 10^{-6} and 10^{-8}. The results are plotted in Fig. 4. In arriving at the curves of Fig. 4, the following system parameters were fixed:

- Effective system temperature, $T_{eff} = 650°K$. This would correspond roughly to a receiver noise figure of 3.5 db and a source temperature of 290°K at the antenna terminals (i.e., $T_{eff} = 290(NF - 1)$, where NF is the receiver noise figure).
- System loss factor, L = 10 db. This factor includes plumbing losses, atmospheric attenuation, and system degradation due to nonoptimum detection processing. This assumed loss factor should be relatively conservative for state-of-the-art phased array radar systems over the frequency region from UHF to X band.
- Azimuth coverage $\Delta\phi$ = 90 deg (1.57 radians).

Referring to the curves of Fig. 4, we shall arbitrarily choose the curves for $\overline{P} \cdot A_e \cdot \sigma = 10^9$ watts $\cdot$ ft^2 $\cdot$ m^2 as being representative of the requirements for long-range surveillance radars. Furthermore, if we limit our attention to the case of a target having a 1-m^2 cross section, the required power aperature product is 10^9 watts $\cdot$ ft^2. The search parameter Γ_s is

$$\Gamma_s = \overline{p}N_tN_r = \frac{4}{\lambda^2}\,\overline{P}A_e = \frac{4}{\lambda^2}\,(10^9)\ . \tag{50}$$

For the frequency interval from 100 Mc to 10 KMc, the corresponding range of values for Γ_s would be

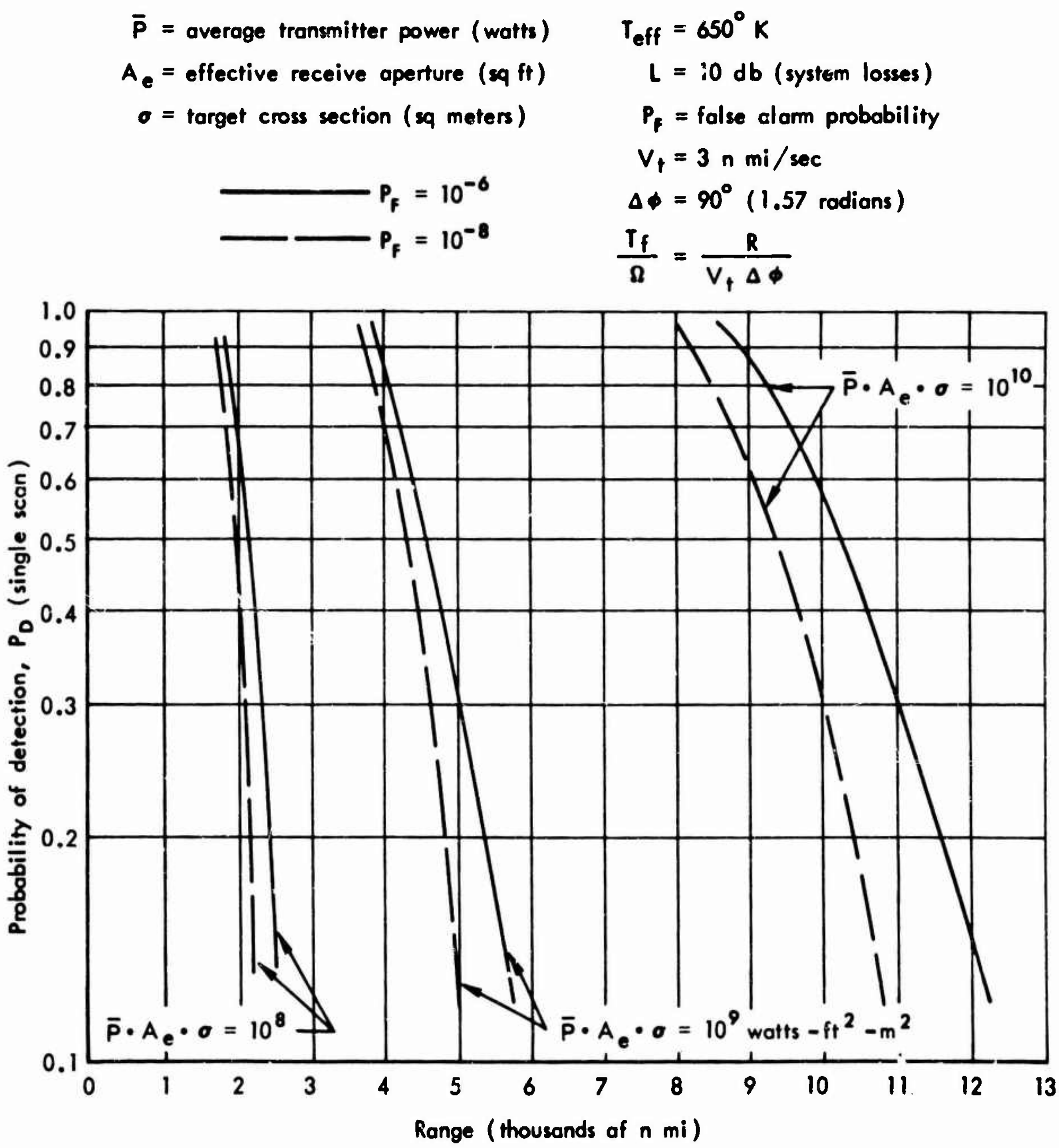


Fig. 4--Detection probability versus range on a single scan

$$4(10^7) \le \Gamma_s = \bar{p}_t N_t N_r \le 4(10^{11}) \; . \tag{51}$$

In order to evaluate the cost and configuration of an optimally configured search array based on Eqs. (27) through (30) of Section IV, it is necessary to select values for the coefficients C_t, C_p, C_r, and $\bar{p}_t$. There is not available a sufficient quantity of phased array research and development program cost data on which estimates of the above coefficients can be based. Hence, rather than decide on specific values for these coefficients, a parametric approach will be used whereby each coefficient is allowed to vary over a range interval. The intervals chosen are based on our own investigations and on discussions with personnel of several organizations engaged in phased array development, and are believed to be representative of current costs and capabilities.

Accordingly the following ranges of coefficients were chosen:

$$\left\{\begin{array}{l} \$100 \le C_r \le \$2000 \; , \\ \$100 \le C_t \le \$4000 \; , \\ \$5 \quad \le C_p \le 15 \;\$/\text{watt} \; , \\ \quad 1 \le \bar{P}_t \le 250 \text{ watts} \; . \end{array}\right. \tag{52}$$

For convenience the optimum cost formula for a search array radar derived in Eq. (29) is rewritten below:

$$C_{\substack{\text{opt} \\ \text{search}}} = 2\left[C_r\left(\frac{C_t}{\bar{p}_t} + C_p\right)\Gamma_s\right]^{\frac{1}{2}} \; . \tag{29}$$

Because of the large number of variables, this result can be presented in several formats. For example, in Fig. 5, by fixing Γ_s, C_p, and $\bar{p}_t$, $C_{\substack{opt\\search}}$ is plotted as a function of C_t for each of several values of C_r. The choice of $\Gamma_s = 10^9$ in Fig. 5 would correspond to a frequency of approximately 500 Mc (i.e., $\lambda \cong 2$ ft in Eq. (50)) in order to achieve the search performance in Fig. 4 for $\overline{PA}_e = 10^9$ watts $\cdot$ ft^2. However, the cost curves of Fig. 5, shown for a fixed Γ_s, would apply to arrays designed at other frequencies and having different power aperture products and hence different search performance, provided λ and $\overline{PA}_e$ ... satisfy Eq. (50).

Referring to Eq. (29), we see that perhaps a more useful form of presentation is shown in Fig. 6, where $C_{\substack{opt\\search}}$ is plotted as a function of the combined variable $[(C_t/\bar{p}_t) + C_p]$. By combining C_t, $\bar{p}_t$, and C_p, these curves are applicable to a variety of cases since only Γ_s is fixed. Furthermore, on log-log paper the cost curves are straight lines so that additional curves can be easily added by evaluating a single new cost point (for each new curve) using Eq. (29) and a reference point on one of the existing curves.

The influence of Γ_s is shown in Fig. 7, where $C_{\substack{opt\\search}}$ is plotted versus Γ_s for each of several sets of values of $[(C_t/\bar{p}_t) + C_p]$ and C_r. Since these curves are also straight lines on log-log paper, the same general comments made with reference to Fig. 6 will apply.

While the curves of Figs. 5 through 7 indicate the optimum hardware cost as a function of both performance (i.e., Γ_s) and the several cost coefficients, they do not show the influence of changes in the cost coefficients on the resulting array configurations.

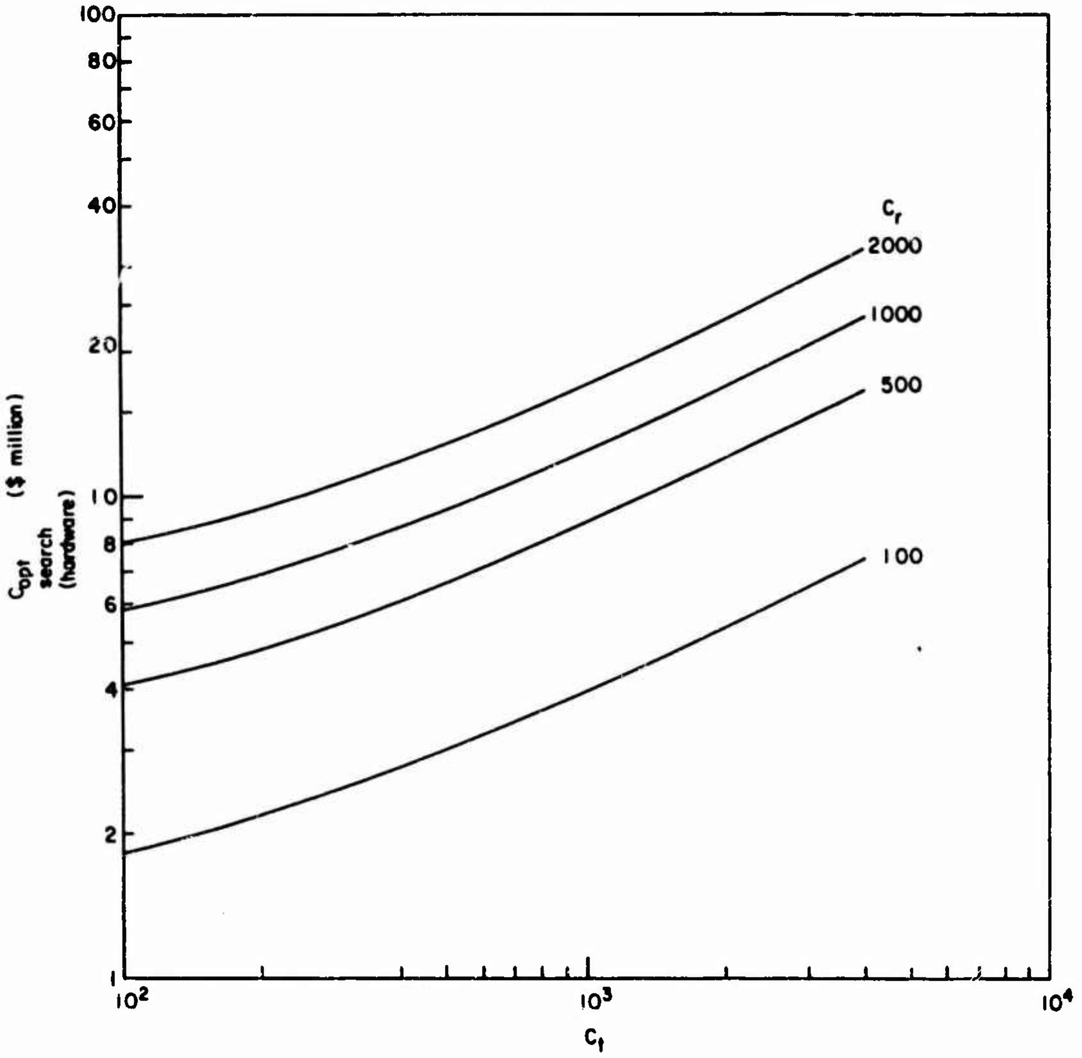


Fig. 5--Optimum search radar cost (C_p = 5, $\bar{P}_t$ = 30, Γ_s = 10^9)

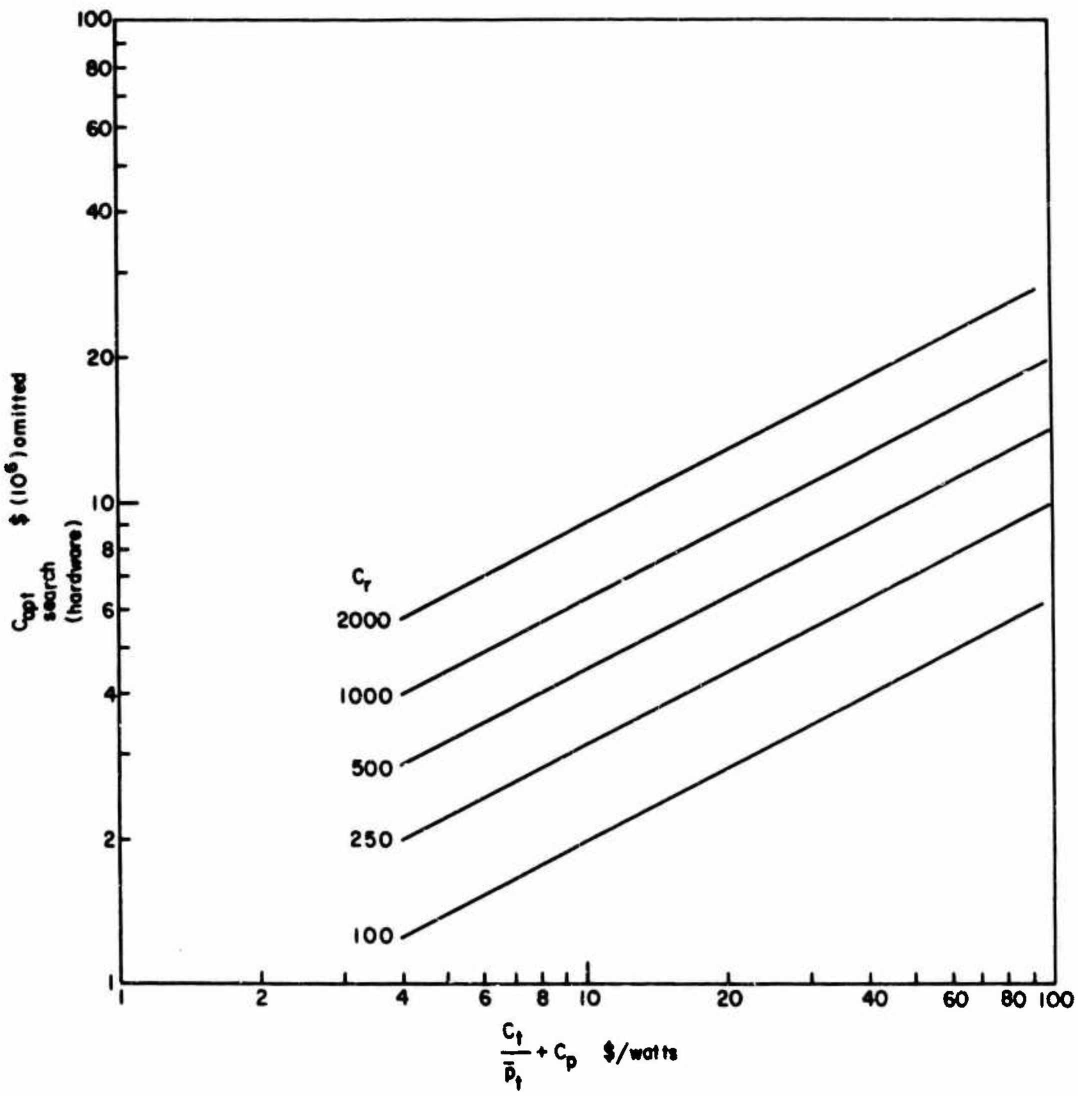
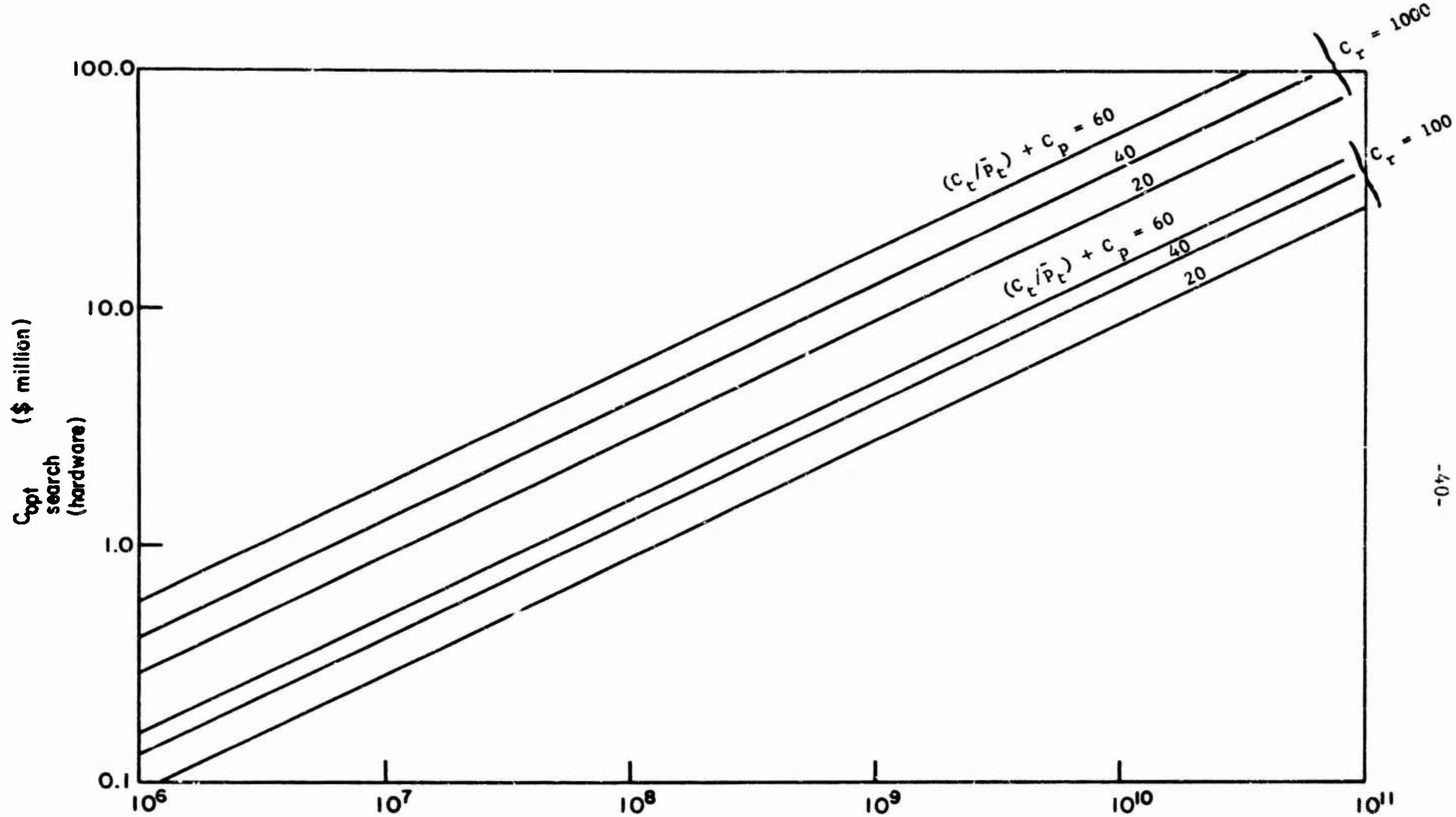


Fig. 6--Optimum search radar cost ($\Gamma_s = 10^9$)



$$\Gamma_s = \bar{p}_t N_t N_r = 16\pi k T_{eff} L \left(\frac{R^4}{\lambda^2 \sigma}\right)\left(\frac{\Omega}{T_f}\right)\left(\frac{E}{N_0}\right)$$

Fig. 7--Optimum search radar cost versus Γ_s

To exhibit these effects, four separate sets of cost coefficients will be chosen to correspond to four distinct module design approacnes arising from the choice of low power versus high power modules in conjunction with narrow band versus wide band operation. The influence of bandwidth on the cost coefficients is exhibited by multiplying C_r and C_t by a factor of two in going from narrow band to wide band operation. If, as has been suggested* in the case of high power transmitters, module cost is proportional to bandwidth to the one-fourth power, then the factor of two in cost would correspond to a percentage bandwidth change by a factor of 16** (e.g., a narrow band case of 0.6 percent bandwidth versus a wide band case of 10 percent bandwidth).

Case 1. Narrow Band Array--Low Power Module (UHF - L Band)

Let $\bar{P}_t$ = 30 watts, C_t = \$1100, C_r = \$1000, C_p = \$5/watt.

Let

$$\Gamma_s = \frac{4(\bar{P}A_e)}{\lambda^2} = 10^9 \quad \text{watts} .$$

For f = 440 Mc, λ = 2.24 ft; hence $\bar{P}A_e = 1.25(10^9)$. From Eq. (27a),

$$N_{t_{opt}} = \left[\frac{\Gamma_s}{\bar{P}_t} \cdot \frac{C_r}{(C_t + C_p\bar{P}_t)}\right]^{\frac{1}{2}} = \sqrt{\frac{10^9}{30} \cdot \frac{10^3}{(1100 + 5(30))}} = 5160.$$

Using Eq. (26),

$$N_{r_{opt}} = N_{r_{opt}} \left(\frac{C_t + C_p\bar{P}_t}{C_r}\right) = 5160\left(\frac{1100 + 5(30)}{1000}\right) = 6450.$$

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*N. E. Feldman (The RAND Corporation), private communication.

**Phasing may also add to the module cost if real time delay scanning is required. In that case, the one-fourth power cost versus bandwidth dependence would be altered.

Finally, from Eq. (28),

$$C_{\substack{\text{opt}\\ \text{search}\\ \text{(hardware)}}} = 2(C_t + C_p\bar{p}_t)N_{t_{\text{opt}}} = 2C_rN_{r_{\text{opt}}}$$

$$= 2(1100 + 5(30))5160 = \$12.9(10^6)\ .$$

This cost figure could have been obtained from either Fig. 5 or Fig. 6, but Fig. 6 is the more useful since it requires fewer assumed fixed parameters. To use Fig. 6, we must first evaluate the additional combined parameter

$$\left(\frac{C_t}{\bar{p}_t} + C_p\right) = \frac{1100}{30} + 5 = 41.7.$$

If we were interested in extrapolating the optimum radar cost for different values of the search performance parameter Γ_s, then Fig. 7 would be the most useful tool. In this case it would be necessary to extrapolate between the several specific values of $[(C_t/\bar{p}_t) + C_p]$ used in Fig. 7.

Case 2. Narrow Band Array--High Power Module (UHF - L Band)

Let $\bar{p}_t$ = 250 watts, C_t = \$3750, C_r = \$1000, C_p = \$5/watt, $\Gamma_s = 10^9 = 4\bar{P}A_e/\lambda^2$. Comparing these values for $\bar{p}_t$ and C_t with those for Case 1, and assuming

(53) $$C_t \propto \bar{p}_t^{\,n}\ ,$$

we have

$$\frac{3750}{1100} = \left(\frac{250}{30}\right)^n\ ,$$

from which we find

$$n = 0.58 \ . \tag{54}$$

The constant of proportionality in Eq. (53) may now be found; thus

$$C_t = K\bar{p}_t^n \ , \tag{55}$$

$$3750 = K(250)^{0.58} \ ,$$

$$K = 153 \ . \tag{56}$$

Combining Eqs. (54) through (56), we arrive at the following equation, which we can use to relate narrow band transmitter module cost to average power:

$$C_t = 153(\bar{p}_t)^{0.58} \quad \$, \text{ with } \bar{p}_t \text{ in watts} \ . \tag{57}$$

For the case at hand we have from Eq. (27a),

$$N_{t_{opt}} = \left[\frac{\Gamma_s}{\bar{p}_t} \frac{C_r}{(C_t + C_p\bar{p}_t)}\right]^{\frac{1}{2}} = \sqrt{\frac{10^9}{250} \frac{1000}{(3750 + 5(250))}} = 894 \ .$$

From Eq. (26),

$$N_{r_{opt}} = N_{t_{opt}}\left(\frac{C_t + C_p\bar{p}_t}{C_r}\right) = 894\left(\frac{3750 + 5(250)}{1000}\right) = 4470 \ .$$

And from Eq. (28),

$$C_{\substack{opt\\ search\\ (hardware)}} = 2(C_t + C_p\bar{p}_t)N_{t_{opt}} = 2(5000)894 = \$8.94(10^6) \ .$$

This cost figure can be independently verified by using either Fig 6 or Fig. 7. First we evaluate the combined parameter

$$\frac{C_t}{\bar{P}_t} + C_p = \frac{3750}{250} + 5 = 20 \ .$$

Using this value, along with the given values of C_r and Γ_s in Figs. 6 and 7, yields the desired cost check.

C. 3. Wide Band--Low Power Module (L-X Band)

For "wide bandwidth" operation, as indicated earlier, both the transmitter and receiver cost coefficients will be multiplied by a factor of two.

Thus, let $\bar{P}_t$ = 110 watts, $\Gamma_s = 10^9$, C_r = \$2000, C_p = \$5/watt. In Eq. (57) we replace K by K' = 2K; hence

$$(58) \quad \begin{cases} C_t = K'(\bar{P}_t)^{0.58} = 306 \cdot (110)^{0.58} = 4680 \ , \\ \\ K' = 2(153) = 306. \end{cases}$$

Thus using Eqs. (27a), (26), and (28), we find

$$N_{t_{opt}} = \sqrt{\frac{10^9}{110} \frac{2000}{(4680 + 5(110))}} = 1864 \ ,$$

$$N_{r_{opt}} = \frac{5230}{2000} 1864 = 4870 \ ,$$

$$C_{\substack{opt \\ search \\ (hardware)}} = 2N_{r_{opt}} C_r = 2(4870)2000 = \$19.5(10^6) \ .$$

Again we evaluate the parameter

$$\frac{C_t}{\bar{p}_t} + C_p = \frac{4680}{110} + 5 = 47.5 \ .$$

Figure 6 provides the simplest check on the above cost using the curve for $C_r = \$2000$.

Case 4. Wide Band--High Power Module (L-X Band)

Let $\bar{p}_t = 500$ watts, $C_r = \$2000$, $C_p = \$5/\text{watt}$. Once again we choose as our performance reference value $\Gamma_s = 10^9$. Here

$$C_t = \$2(153)\bar{p}_t^{\,0.58} = 306(500)^{0.58} = \$11,275 \ .$$

Thus, following the same procedure as in the previous cases, we find

$$N_{t_{opt}} = \sqrt{\frac{10^9}{500} \frac{2000}{(11,275 + 5(500))}} = 538 \ ,$$

$$N_{r_{opt}} = \frac{13,775}{2000}(538) = 3708 \ ,$$

$$C_{\substack{opt\\ \text{search}\\ \text{(hardware)}}} = 2C_r N_{r_{opt}} = 2(C_t + C_p\bar{p}_t)N_{t_{opt}} = 2(2000)3708$$

$$= \$14.8(10^6) \ .$$

As a check on this cost, once again we refer to Fig. 6 after evaluating the parameter

$$\left(\frac{C_t}{\bar{p}_t} + C_p\right) = \frac{11,275}{500} + 5 = 27.5 \ .$$

Using this parameter along with the curve for C_r = \$2000, the above cost is verified.

TRACKING RADAR

In this subsection, the optimum design and cost of several tracking arrays will be presented. Starting with an assumed set of tracking system requirements, the tracking parameter of Eq. (20) can be evaluated, and the tracking design and cost formulas derived in Eqs. (35), (36), and (37) will be evaluated for the sets of coefficients ($\bar{p}_t$, C_t, C_r, C_p) corresponding to each of the four cases previously presented.

Tracking System Parameters

For purposes of illustration the radar will be required to track a target having a cross section of one m^2 with an angle accuracy of 0.005 radian (approximately 0.3 deg) employing a tracking dwell time of 60 seconds for a slant range of 3000 n mi. The pertinent system parameters are summarized below:

$$R = 3000 \text{ n mi}, \qquad T_d = 60 \text{ seconds},$$
$$\sigma = 1 \text{ m}^2, \qquad L = 10 \text{ db},$$
$$\sigma_{\theta_x} = \sigma_{\theta_y} = 0.005 \text{ radian}, \qquad T_{eff} = 650^\circ\text{K}.$$

Substituting these specific requirements into Eq. (20), we find the tracking parameter Γ_t to be

$$(59) \qquad \Gamma_t = \bar{p}_t N_t^2 N_r^2 = \frac{384 R^4 k T_{eff} L}{\pi \sigma \lambda^2 \sigma_{\theta_x} \sigma_{\theta_y} T_d} = \frac{7.26(10^{13})}{\lambda^2} \text{ watts.}$$

For differing values of the various system parameters, the above value of Γ_t can easily be scaled. Taking into account what are felt to be reasonable intervals of variation for each of the variables in the equation for Γ_t, the following range of values of Γ_t should include most realistic sets of system requirements:

(60) $$10^{10} \le \Gamma_t = \bar{p}_t N_t^2 N_r^2 \le 10^{18} \quad .$$

If we refer to Eq. (37), a reasonable choice for presenting tracking cost curves is to plot $C_{\substack{opt\\track}}$ versus the combined parameter $[(C_t + C_p\bar{p}_t)/(\bar{p}_t)^{\frac{1}{2}}]$ for each of several values of C_r and a fixed Γ_t. These curves are plotted in Fig. 8 for $\Gamma_t = 10^{14}$; for C_r = \$100, \$250, \$500, \$1000, \$2000; and for

$$50 \le \left[\frac{C_t + C_p\bar{p}_t}{(\bar{p}_t)^{\frac{1}{2}}}\right] \le 1000 \quad \$/(\text{watt})^{\frac{1}{2}}$$

Figure 9 similarly presents $C_{\substack{opt\\track}}$ versus Γ_t for each of several combinations of C_r and the parameter

$$\left[\frac{C_t + C_p\bar{p}_t}{(\bar{p}_t)^{\frac{1}{2}}}\right].$$

Once again to illustrate the influence of specific sets of cost coefficients on the resulting optimum physical design (i.e., choice of N_t and N_r) and the optimum tracking cost, the same four cases previously considered for search will be reevaluated for a tracking radar assuming $\Gamma_t = 10^{14}$.

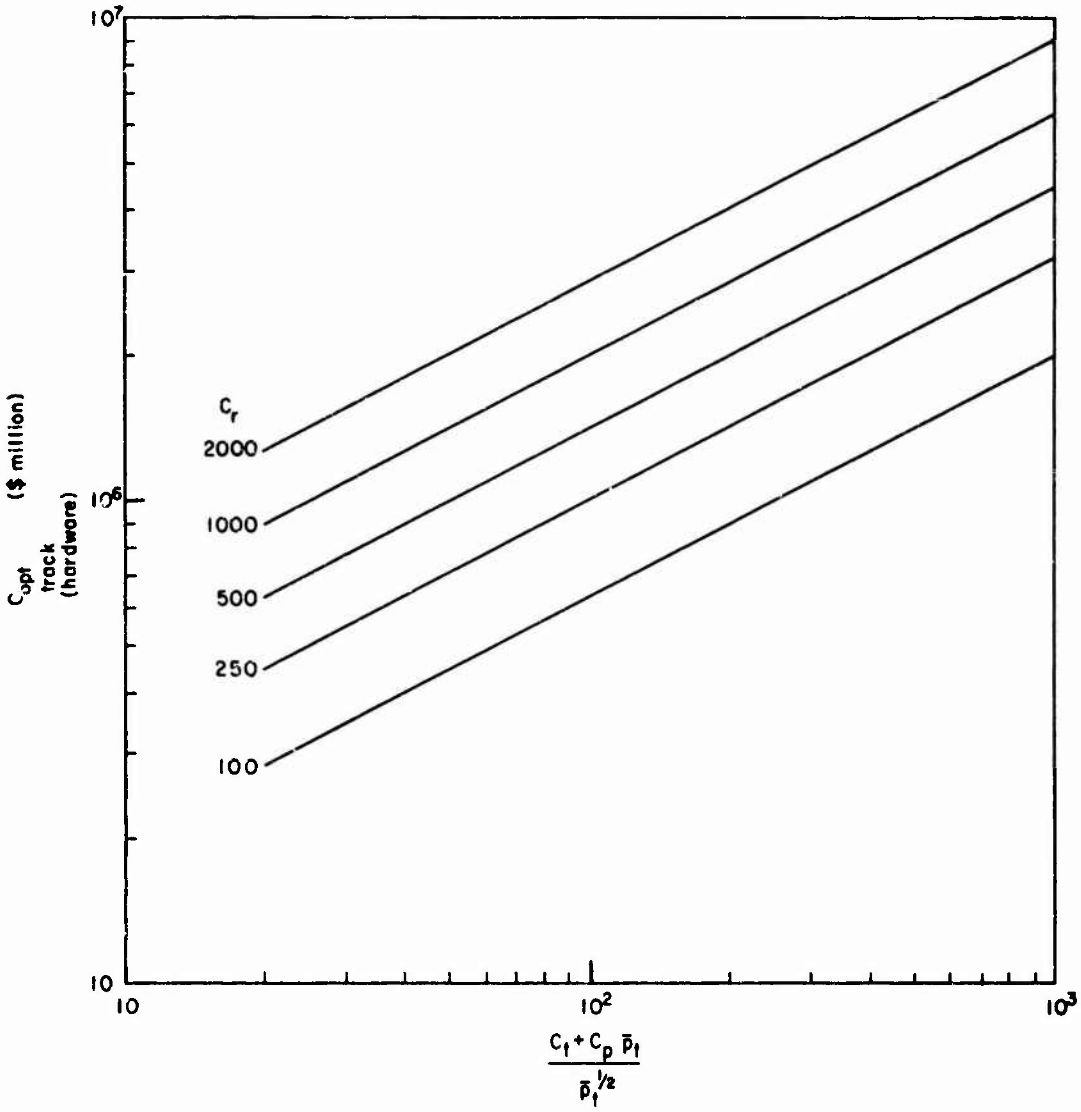
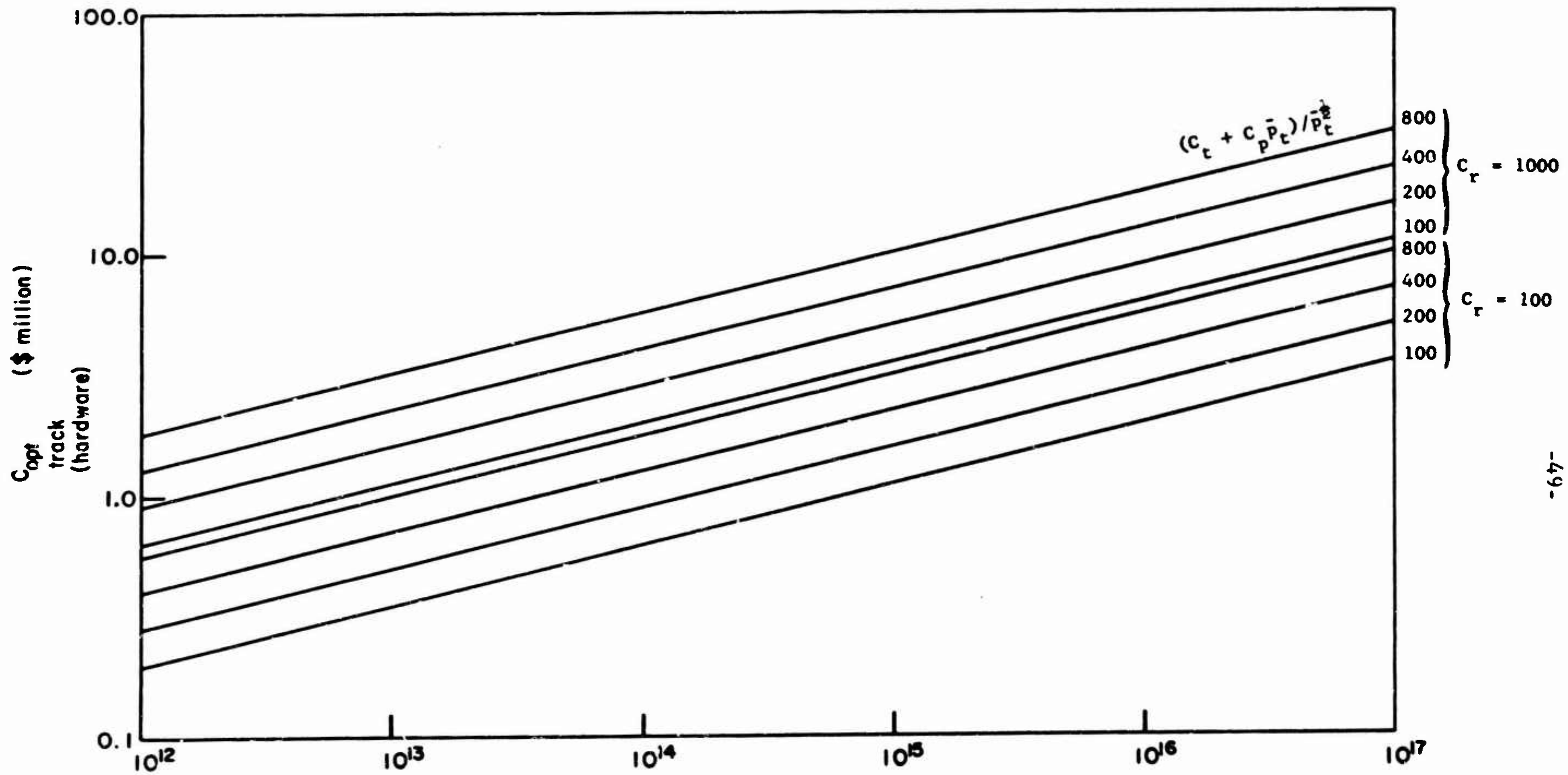


Fig. 8--Optimum tracking radar cost ($\Gamma_t = 10^{14}$)



$$\Gamma_t = \bar{P}_t N_t^2 N_r^2 = \frac{384}{\pi} \frac{R^4}{\sigma\lambda^2} \frac{kT_{eff}L}{\sigma_{\theta_x}\sigma_{\theta_y}} \frac{1}{T_d}$$

Fig. 9--Optimum tracking radar cost versus Γ_t

Case 1. Narrow Band Array--Low Power Module (UHF - L Band).

Let $\bar{p}_t$ = 30 watts, C_t = \$1100, C_r = \$1000, C_p = \$5/watt, $\Gamma_t = 10^{14}$, and f = 1160 Mc (L band).

From Eq. (36a),

$$N_{t_{opt}} = \left[\frac{\Gamma_t}{\bar{p}_t} \frac{C_r^2}{(C_t + C_p\bar{p}_t)^2}\right]^{\frac{1}{2}} = \left[\frac{10^{14}}{30} \frac{10^6}{(1100 + 150)^2}\right]^{\frac{1}{2}} = 1208 \ .$$

Using Eq. (35),

$$N_{r_{opt}} = \left(\frac{C_t + C_p\bar{p}_t}{C_r}\right) N_{t_{opt}} = \frac{1250}{1000} (1208) = 1510 \ .$$

Next evaluate

$$\left[\frac{C_t + C_p\bar{p}_t}{(\bar{p}_t)^{\frac{1}{2}}}\right] = \frac{1100 + 150}{\sqrt{30}} = 228 \ .$$

Reference to Fig. 8 or Eq. (37) yields the optimum tracking radar cost:

$$C_{opt_{track}} = 2(C_t + C_p\bar{p}_t)N_{t_{opt}} = \$3.02(10^6) \ .$$

For reference to the forthcoming discussion, the available search parameter Γ_s for this tracking array will also be evaluated:

$$\Gamma_s = \bar{p}_t N_t N_r = 30(1.208)(1.51)10^6 = 5.47(10^7) \ .$$

The corresponding available power aperture product is

$$\overline{PA}_e = \frac{\lambda^2\Gamma_s}{4} = \frac{(0.85)^2(5.4710^7)}{4} = 9.95(10^6) \quad \text{watts} \cdot \text{ft}^2 \ .$$

This available power aperture is down by a factor of over 100 from the 10^9 watts · ft^2 required for the search performance previously discussed. Hence this radar operating in a search mode would have a range capability of

$$R_d = 3000 \cdot \left[\frac{9.95(10^6)}{10^9}\right]^{\frac{1}{4}} = 950 \quad \text{n mi}$$

(see Eq. (3)), assuming the other system parameters to be unchanged.

Case 2. Narrow Band Array--High Power Module (UHF - L Band)

Let $\bar{p}_t$ = 250 watts, C_t = \$3750, C_r = \$1000, C_p = \$5/watt, Γ_t = 10^{14}, and f = 1160 Mc. From Eqs. (36a) and (35),

$$N_{t_{opt}} = \left[\frac{10^{14}}{250} \frac{10^6}{[3750 + 5(250)]^2}\right]^{\frac{1}{4}} = 356$$

and

$$N_{r_{opt}} = \frac{[3750 + 5(250)]}{1000} \cdot 356 = 1780 \ .$$

Next evaluate

$$\left[\frac{C_t + C_p\bar{p}_t}{(\bar{p}_t)^{\frac{1}{2}}}\right] = \frac{3750 + 5(250)}{\sqrt{250}} = 316 \ .$$

From Fig. 8 or Eq. (37),

$$C_{opt_{track}} = 2(C_t + C_p\bar{p}_t)N_{t_{opt}} = \$3.56(10^6) \ .$$

Once again the corresponding available search parameter Γ_s is

$$\Gamma_s = \bar{p}_t N_t N_r = 250(356)1780 = 1.58(10^8) \ .$$

The available power aperture product is

$$\overline{P}A_e = \frac{\lambda^2}{4}\Gamma_s = \frac{(0.85)^2}{4}\, 1.58(10^8) = 2.88(10^7) \quad \text{watts} \cdot \text{ft}^2 \ .$$

Again this power aperture is less than that required for the specified search performance.

<u>Case 3. Wide Band--Low Power Module (L-X Band)</u>

Let $\bar{p}_t$ = 110 watts, C_t = \$4680, C_r = \$2000, C_p = \$5/watt, Γ_t = 10^{14}, and f = 1160 Mc. From Eqs. (36a) and (35),

$$N_{t_{opt}} = \left[\frac{10^{14}}{110}\,\frac{4(10^6)}{(4680 + 5(110))^2}\right]^{\frac{1}{4}} = 602 \ ,$$

$$N_{r_{opt}} = \frac{(4680 + 550)}{2000} \cdot 602 = 1576 \ .$$

Next evaluate

$$\left[\frac{C_t + C_p\bar{p}_t}{(\bar{p}_t)^{\frac{1}{2}}}\right] = \frac{5230}{(110)^{\frac{1}{2}}} = 498 \ .$$

From Fig. 8 or Eq. (37),

$$C_{opt\ track} = 2(C_t + C_p\bar{p}_t)N_{t_{opt}} = \$6.3(10^6) \ .$$

The available search parameter Γ_s is

$$\Gamma_s = \bar{p}_t N_t N_r = 110(602)1576 = 1.04(10^8)$$

and

$$\overline{PA}_e = \frac{\lambda^2}{4}\Gamma_s = \frac{(0.85)^2}{4} 1.04(10^8) = 1.89(10^7) \quad \text{watts} \cdot \text{ft}^2 .$$

Case 4. Wide Band--High Power Module (L-X Band)

Let $\bar{p}_t$ = 500 watts, C_t = \$11,275, C_r = \$2000, C_p = \$5/watt, Γ_t = 10^{14}, and f = 1160 Mc. From Eqs. (36a) and (35),

$$N_{t_{opt}} = \left[\frac{10^{14}}{500} \frac{4(10^6)}{(11,275 + 2500)^2}\right]^{\frac{1}{4}} = 255 ,$$

$$N_{r_{opt}} = \frac{(11,275 + 2500)}{2000} \cdot 255 = 1760 .$$

Next,

$$\left[\frac{C_t + C_p\bar{p}_t}{(\bar{p}_t)^{\frac{1}{2}}}\right] = \frac{13,775}{(500)^{\frac{1}{2}}} = 616 .$$

From Fig. 8 or Eq. (37),

$$C_{\substack{opt\\track}} = 2(C_t + C_p\bar{p}_t)N_{t_{opt}} = \$7.02(10^6) .$$

The available search parameter Γ_s is

$$\Gamma_s = \bar{p}_t N_t N_r = 500(255)1760 = 2.24(10^8)$$

and

$$\overline{PA}_e = \frac{\lambda^2}{4}\Gamma_s = \frac{(0.85)^2}{4}(2.24 \times 10^8) = 4.07(10^7) \quad \text{watts} \cdot \text{ft}^2 .$$

COST COMPARISONS

As an aid to the discussion, the optimum cost formulas for search and track derived in Eqs. (29) and (37) are rewritten here in modified form. Thus

$$(29) \qquad C_{\substack{opt\\search}} = 2\Gamma_s^{\frac{1}{2}}\left[C_r\left(\frac{C_t}{\bar{P}_t} + C_p\right)\right]^{\frac{1}{2}},$$

and

$$(37) \qquad C_{\substack{opt\\track}} = 2\Gamma_t^{\frac{1}{2}}\left[C_r\left(\frac{C_t}{\bar{P}_t} + C_p\right)\bar{P}_t^{-\frac{1}{2}}\right]^{\frac{1}{2}}.$$

Equation (29) shows that the cost for an optimum search array radar is proportional to the square root of the total cost per average watt for the transmitter (assuming fixed values for C_r and Γ_s). Hence, as was indicated in the four search design cases presented, <u>use of higher power modules with lower cost per watt results in less expensive search arrays</u>.

However, <u>in the case of the tracking array design</u>, the situation is reversed, so that <u>the use of higher power modules</u> (with less cost per average watt) <u>results in higher optimum cost tracking arrays</u> (e.g., compare Case 1 with 2 or Case 3 with 4). This may be explained as follows: In Eqs. (55) through (57) the transmitter module cost is related to average power by the relation

$$C_t = K(\bar{p}_t)^n, \quad n = 0.58 > \tfrac{1}{2}.$$

Substituting this result into Eq. (37) yields

$$C_{\substack{opt\\track}} = 2\Gamma_t^{\frac{1}{2}}\left[C_r\left(K(\bar{p}_t)^{n-\frac{1}{2}} + C_p(\bar{p}_t)^{\frac{1}{2}}\right)\right]^{\frac{1}{2}} . \tag{61}$$

From Eq. (61) it is clear that the cost of the optimum tracking radar will be an increasing function of $\bar{p}_t$ for all $n \geq 1/2$; and this condition is satisfied for the value $n = 0.58$ used in the cost example (see Eq. (57)).

PERFORMANCE COMPARISONS

In the discussion of the four tracking designs it was shown that in each case the tracking designs would not have sufficient capability to achieve the required search performance previously stipulated. It follows conversely that each of the search array designs would have more than sufficient capability to achieve the required tracking performance (i.e., the available Γ_t exceeds the required value for each of the search cases).

However, contrary to what one might expect, the above result is not part of some more general theorem but results rather from the combination of the specific tracking and search requirements and the limited range of values of $\bar{p}_t$ considered. The following discussion should help clarify the conditions leading to the above result.

From Eqs. (8) and (20) the search and track parameters for an array radar may be rewritten as

$$\Gamma_{s_{req}} = 16\pi R^4 \left(\frac{kT_{eff}L}{\lambda^2\sigma}\right) \frac{\Omega}{T_f} \frac{E}{N_0} , \tag{8a}$$

(8b) $$\Gamma_{s_{avail}} = \bar{p}_t N_t N_r \ ,$$

(20a) $$\Gamma_{t_{req}} = \frac{384}{\pi} R^4 \left(\frac{kT_{eff} L}{\lambda^2 \sigma} \right) \frac{1}{\sigma_{\theta_x} \sigma_{\theta_y}} \frac{1}{T_d} \ ,$$

(20b) $$\Gamma_{t_{avail}} = \bar{p}_t N_t^2 N_r^2 \ .$$

In rewriting these equations subscripts have been used to make clear the difference between the required values of Γ_t and Γ_s, resulting from specific performance requirements, and the corresponding available values of Γ_t and Γ_s, which are only related to the values of $\bar{p}_t$, N_t, and N_r.

From Eqs. (8b) and (20b) we have

(62) $$\frac{\Gamma_{t_{avail}}}{\Gamma_{s_{avail}}} = N_t N_r \ ,$$

and from Eqs. (8a) and (20a),

(63) $$\frac{\Gamma_{t_{req}}}{\Gamma_{s_{req}}} = \frac{24}{\pi^2} \frac{\frac{1}{\sigma_{\theta_x} \sigma_{\theta_y}} \cdot \frac{1}{T_d}}{\frac{\Omega}{T_f} \cdot \frac{E}{N_0}} \ .$$

There are two separate cases to be considered, each having two alternative subcases.

<u>Case A</u>

Let

(64) $$N_t N_r = \frac{\Gamma_{t_{avail}}}{\Gamma_{s_{avail}}} \geq \frac{\Gamma_{t_{req}}}{\Gamma_{s_{req}}} \ .$$

A1. Search Radar. For a search radar, $\Gamma_{s_{avail}} = \Gamma_{s_{req}}$ by design. Therefore from Eq. (64),

(65) $$\Gamma_{t_{avail}} \geq \Gamma_{t_{req}} .$$

A2. Tracking Radar. For a tracking radar, $\Gamma_{t_{avail}} = \Gamma_{t_{req}}$ by design. Again from Eq. (64),

(66) $$\Gamma_{s_{avail}} \leq \Gamma_{s_{req}} .$$

Case B

Let

(67) $$N_t N_r = \frac{\Gamma_{t_{avail}}}{\Gamma_{s_{avail}}} \leq \frac{\Gamma_{t_{req}}}{\Gamma_{s_{req}}} .$$

B1. Search Radar. Once again by design, $\Gamma_{s_{avail}} = \Gamma_{s_{req}}$; hence from Eq. (67),

(68) $$\Gamma_{t_{avail}} \leq \Gamma_{t_{req}} .$$

B2. Tracking Radar. By design, $\Gamma_{t_{avail}} = \Gamma_{t_{req}}$, from which Eq. (67) yields

(69) $$\Gamma_{s_{avail}} \geq \Gamma_{s_{req}} .$$

It is clear that for the specific set of search and track requirements given in the illustrative examples, the situation corresponds to Case A. Referring to Eqs. (50) and (59), we see that

$$(70) \qquad \frac{\Gamma_{t_{req}}}{\Gamma_{s_{req}}} = \frac{7.26(10^{13})/\lambda^2}{4(10^9)/\lambda^2} = 1.82(10^4) .$$

The same result could be obtained directly from Eq. (63), which also exhibits the fact that this ratio is independent of the wavelength λ. For each of the search array designs, the choice of $\bar{p}_t$ resulted in a value of $N_t N_r$ such that the inequality of Case A (Eq. (64)) was satisfied, leading to the conclusions of Eqs. (65) and (66). However, by a sufficient increase in tracking requirements and/or a decrease in search requirements, the ratio in Eq. (70) could be increased so that the inequality of Case B (Eq. (67)) would be satisfied, resulting in the conclusions of Eqs. (68) and (69).

Clearly, if the same radar is to be capable of alternately performing either the tracking or search functions, Cases A1 and B2 are the only acceptable alternatives. It is often possible to achieve the operational flexibility afforded by Cases A1 and B2 without overdesigning the radar for its primary function by either of the following approaches:

- For a search radar, the average power per module, $\bar{p}_t$, should be chosen sufficiently low to ensure the inequality of Eq. (64), i.e., $\Gamma_{t_{avail}} \geq \Gamma_{t_{req}}$.
- For a tracking radar, the average power per module should be chosen sufficiently high to ensure the inequality of Eq. (67), i.e., $\Gamma_{s_{avail}} \geq \Gamma_{s_{req}}$.

However, this approach to design flexibility must be paid for by increased cost since the use of either lower power modules for search or higher power modules for tracking results in an increase in the optimum hardware cost as shown in the discussion.

NONOPTIMUM DESIGNS

Reduced Transmitter Power

To illustrate the cost penalty incurred if for some reason the optimum design configuration is not employed, consider the search array design of Case 2. Suppose the available average power per transmitter module is reduced by a factor of 10 and the cost coefficients remain unchanged. This fact may not become evident until after the initial optimum configuration is designed, and it may not be feasible to reoptimize the design.* In such a case, a nonoptimum design that achieves the same required Γ_s would be represented by increasing N_t tenfold to account for the reduced $\bar{p}_t$. Thus $\bar{p}_t = 25$ watts, $C_t = \$3750$, $C_r = \$1000$, $C_p = \$5/\text{watt}$, and $\Gamma_s = 10^9$. Then

$$N_t = 10(894) = 8940 ,$$

$$N_r = 4470 \text{ (same as Case 2)} ,$$

$$\Gamma_s = \bar{p}_t N_t N_r = 25(8940)4470 = 10^9 ,$$

$$C_{\substack{\text{nonopt}\\\text{search}\\\text{(hardware)}}} = (C_t + (p\bar{P}_t)N_t + C_r N_r$$

$$= (3750 + 125)8940 + 4470(10^3) = \$39.1(10^6) .$$

This must be compared with the optimum design cost of $\$8.94(10^6)$ for

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*A possible example might be the decision to reduce the average tube power to increase life expectancy.

Case 2. If one were to reoptimize the design in this case, based on the new value of $\bar{p}_t$, then from Eq. (29) the new optimum cost would be

$$C_{\substack{\text{opt}\\\text{search}\\\text{(hardware)}}} = \left[2\Gamma_s C_r \frac{(C_t + C_p \bar{p}_t)}{\bar{p}_t}\right]^{\frac{1}{2}}$$

$$= 8.94\left(\frac{155}{20}\right)^{\frac{1}{2}} = \$24.9(10^6) \ .$$

Combined Array

As a second example of a nonoptimum design, consider the effect on overall hardware cost of using a single combined array in which each element is used both for transmitting and receiving. Again we use the search radar of Case 2 as our point of reference.

Thus let $\bar{p}_t$ = 250 watts, C_{t+r} = 3750 + 1000 = \$4750, C_p = \$5/watt, and $\Gamma_s = 10^9$. Since

$$N_t = N_r = N \ ,$$

$$\Gamma_s = \bar{p}_t N_t N_r = \bar{p}_t N^2 = 10^9 \ .$$

Solving for N,

$$N = \left(\frac{10^9}{250}\right)^{\frac{1}{2}} = 2000 \ ,$$

$$C = (C_p \bar{p}_t + C_{t+r})N = (5(250) + 4750)2000 = \$12(10^6) \ .$$

The cost of this combined array design is 34 percent greater than the \$8.94(10^6) for the optimum configuration of Case 2 using separate transmit and receive arrays. The practice of simply adding the separate transmitter and receiver module costs to obtain the cost of the combined module is reasonable since, although the antenna element and perhaps the phase shifter may be shared, the resulting cost savings would tend to be offset by the required use of a duplexer (in the combined module), which is usually costly and may also tend to increase system losses.

Simultaneous Search and Track

It is often advantageous to have a simultaneous search and track capability using a single radar. The same transmitting and receiving arrays can be used for both functions, and the total radiated energy required for both functions can be time shared so that the total average power would be additive.

Since for the examples presented the search designs were found to have more than sufficient available Γ_t to alternately perform the tracking function, it is clear that the proper design approach to achieving a simultaneous capability is to modify one of the search designs to allow for the sharing of total average power.

Consider the search array design of Case 2. For the specified search performance, the required power aperture product was found to be $\overline{P}A_e = 10^9$ watts $\cdot$ ft^2. From Eq. (50) the search parameter is

$$\Gamma_s = \frac{4\overline{P}A_e}{\lambda^2} = \frac{4(10^9)}{\lambda^2} \quad . \tag{50}$$

UHF DESIGN

By selecting an operating frequency of 495 Mc, Eq. (50) yields $\Gamma_s = 10^9$, which is the value used in the example of Case 3. Since the tracking function must be performed at the same frequency, Eq. (59) yields for the required value of Γ_t:

$$\Gamma_{t_{req}} = \frac{7.26(10^{13})}{\lambda^2} = 1.82(10^{13}) \quad . \tag{71}$$

Since in Case 2 the total average power per module is 250 watts, we must select a smaller average power for the search function so that

$$\bar{P}_{t_{total}} = \bar{P}_{t_{search}} + \bar{P}_{t_{track}} \leq 250 \quad \text{watts} \quad . \tag{72}$$

From Eqs. (27a) and (27b),

$$N_{t_{opt}} \cdot N_{r_{opt}} = \frac{\Gamma_{s_{req}}}{\bar{P}_{t_{search}}} \quad . \tag{73}$$

Furthermore, the available Γ_t is

$$\Gamma_{t_{avail}} = \bar{P}_{t_{track}} \; N_t^2 N_r^2 \quad .$$

Equating this to the required value of Γ_t (see Eq. (71)) and substituting for $N_t N_r$ from Eq. (73) yields

$$\begin{aligned} \bar{P}_{t_{track}} \; N_t^2 N_r &= \bar{P}_{t_{track}} \; \frac{\Gamma_{s_{req}}}{\bar{P}^{-2}_{t_{search}}} = \Gamma_{t_{req}} \; ; \\ \therefore \; \bar{P}_{t_{track}} &= \frac{\Gamma_{t_{req}}}{\Gamma^2_{s_{req}}} \; \bar{P}^{-2}_{t_{search}} \quad . \end{aligned} \tag{74}$$

For the specific values $\Gamma_{s_{req}} = 10^9$ and $\Gamma_{t_{req}} = 182(10^{13})$, Eq. (74) yields

(75) $$\bar{p}_{t_{track}} = 18.2(10^{-6})\bar{p}^2_{t_{search}} \quad .$$

For example, if $\bar{p}_{t_{search}} = 248$ watts, then Eq. (75) yields

$$\bar{p}_{t_{track}} = 1.1 \text{ watts} \ ,$$

and the total average power is less than 250 watts, as was required in Eq. (72).

If it is desirable to track more than one target, the search power can be reduced to accommodate the added targets while a fixed value for the total average power is maintained.

For example, if $\bar{p}_{t_{search}} = 235$ watts, Eq. (75) yields

$$\bar{p}_{t_{track}} = 18.2(10^{-6})(235)^2 = 1 \text{ watt.}$$

The radar can then track $M = 15$ targets, since

$$\bar{p}_{t_{total}} = \bar{p}_{t_{search}} + M\bar{p}_{t_{track}} = 235 + 15(1) = 250 \text{ watts} \ .$$

To evaluate the cost and design configuration for this radar we have $\bar{p}_{t_{search}} = 235$ watts, $C_t = \$3750$, $C_r = \$1000$, and $C_p = \$5/\text{watt}$. From Eqs. (26) through (28) we obtain

$$N_{t_{opt}} = \left[\frac{10^9}{235} \cdot \frac{1000}{(3750 + 5(235))}\right]^{\frac{1}{2}} = 930 \ ,$$

$$N_{r_{opt}} = \left(\frac{3750 + 5(235)}{1000}\right) 930 = 4580 \ .$$

From Eq. (28),

$$C_{\substack{opt \\ search \\ (hardware)}} = 2(C_t + C_p\bar{P}_t)N_{t_{opt}} = 2(4925)930 = \$9.16(10^6) \ .$$

The added cost to do the tracking is only that incurred in producing the additional average tracking power for the assumed tracking load of M = 15 targets:

$$\bar{P}_{tracking} = N_t M \bar{p}_{t_{tracking}} = 930 \cdot 15 \cdot 1 = 13,950 \quad \text{watts} \ ,$$

$$C_{\substack{add \\ tracking}} = C_p \bar{P}_{tracking} = 5(13,950) = \$69,750 \ .$$

Hence,

$$C_{\substack{total \\ (hardware)}} = 9,160,000 + 69,750 = \$9.23(10^6) \ .$$

This cost for performing simultaneously the search and track functions is only slightly greater than the corresponding cost of $\$8.94(10^6)$ for the optimum search array design of Case 2. This is because the search requirements were dominant for the specific performance requirements used in the examples.

As mentioned in Section IV, we can now show that the cost of the above nonoptimum design differs only slightly from that for the optimum combined search and track array derived in Eqs. (40) through (46).

We start with

$$\Gamma_s = 10^9, \quad \Gamma_t = 15(1.82)10^{13} = 27.3(10^{13}) \;;$$

$$\bar{p}_s = 235, \quad \bar{p}_t = 15 \;;$$

$$C_r = \$1000, \quad C_t = \$3750, \quad C_p = \$5/\text{watt} \;.$$

From Eq. (45a),

$$N_{t_{opt}} = \left\{ \frac{C_r}{2[C_t + C_p(\bar{p}_t + \bar{p}_s)]} \left[\left\{ \left(\frac{\bar{p}_s}{\bar{p}_t} \right)^2 + 4 \frac{(\Gamma_s + \Gamma_t)}{\bar{p}_t} \right\}^{\frac{1}{2}} - \frac{\bar{p}_s}{\bar{p}_t} \right] \right\}^{\frac{1}{2}}$$

$$= \left\{ \frac{10^3}{2[3750 + 5(15+235)]} \left[\left\{ \left(\frac{235}{15} \right)^2 + 4 \left(\frac{10^9 + 27.3(10^{13})}{15} \right) \right\}^{\frac{1}{2}} - \frac{235}{15} \right] \right\}^{\frac{1}{2}} ,$$

$$N_{t_{opt}} = 920 \;.$$

From Eq. (43),

$$N_{r_{opt}} = \frac{[C_t + C_p(\bar{p}_t + \bar{p}_s)]}{C_r} N_{t_{opt}} = \frac{(3750) + 5(15+235))}{1000} \cdot (920) = 4610 \;.$$

Finally,

$$C_{opt_{combined}} = [C_t + C_p(\bar{p}_t + \bar{p}_s)]N_{t_{opt}} + C_r N_{r_{opt}}$$

$$= 5(10^3)920 + 10^3(4610) = \$9.22(10^6) \;.$$

This optimum cost is only slightly less than the $\$9.23(10^6)$ arrived at for the nonoptimum case. The numbers of transmitter and receiver elements are also only slightly different.

Simultaneously combining the search and track functions will, of course, incur additional signal processing cost. However, this would not be reflected in the above cost figures since signal processing cost was not included in the hardware cost.

VI. ARRAY THINNING

THINNED ARRAYS

During the past several years a number of investigators have studied the effect of varying element spacing on the performance of linear and two-dimensional arrays.* The results of these studies have confirmed the intuitive feeling that by proper choice of element positions, it should be possible to increase the average element spacing (and hence use fewer elements for a given size aperture) without appreciably degrading the antenna's performance. The net effect is a reduction in cost without an ostensible change in performance.

The results of the studies to date can be summarized roughly as follows:

1. The gain and effective aperture of a thinned array are proportional to the number of elements in the array and do not depend on the extent of the physical aperture.

2. The beamwidth in any plane is primarily determined by the linear extent of the aperture in this plane and not by the number of array elements.

3. To attain a specified side lobe level, a thinned array must contain more than some minimum number of elements. This minimum

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*Y. T. Lo, "High Resolution Antenna Arrays with Randomly Spaced Elements," 1963 PTGAP International Symposium, Space Telecommunications, July 1963, pp. 212-218; A. Ishimaru and Y. Chen, "Broadband Antenna Arrays by Unequal Spacings," 1963 PTGAP International Symposium, Space Telecommunications, July 1963, pp. 219-223.

number, which depends on the illumination taper, increases only very slowly as the array is thinned very rapidly.

The fact that gain and solid angle of a thinned array may be treated as independent parameters (as opposed to the functional relation of Eq. (4a) for a filled array) will now be used to re-evaluate the design of search and track array radars.

SEARCH

From Eq. (2) the single scan signal-to-noise ratio in search is

$$\frac{E}{N_0} = \frac{\overline{P}G_t A_e \sigma}{(4\pi)^2 R^4 k T_{eff} L} T_d \,. \tag{2}$$

In the general case where there may be separate transmit and receive arrays (each of which it may be desirable to thin), we have

$$\left\{\begin{array}{lll}
\text{(a)} & A_{er} & \cong N_r \dfrac{\lambda^2}{4} \,, \\[2ex]
\text{(b)} & G_t & = \dfrac{4\pi A_{et}}{\lambda^2} \cong N_t \pi \,, \\[2ex]
\text{(c)} & \overline{P} & = \overline{p}_t N_t \,, \\[2ex]
\text{(d)} & T_d & = T_f \dfrac{\omega_t}{\Omega} \,, \\[2ex]
\text{(e)} & \omega_t & \cong \dfrac{\lambda^2}{L_{x_t} L_{y_t}} = \dfrac{1}{\mathcal{L}_{x_t} \mathcal{L}_{y_t}} \,,
\end{array}\right. \tag{76}$$

where L_{x_t} and L_{y_t} are two physical aperture dimensions in orthogonal directions, $\mathcal{L}_{x_t}$ and $\mathcal{L}_{y_t}$ are the corresponding normalized transmitter aperture dimensions, and ω_t is the solid angle of the transmitted beam.

While Eqs. (76) apply generally to arrays that are either filled or thinned, the actual constants of proportionality in (a), (b), and (c) depend on the average element spacing and aperture illumination taper and may differ somewhat from those indicated. For the purposes of this study, such differences will be ignored.

Substitution of (76) into (2) yields

$$(77) \qquad \Gamma_s = \bar{p}_t N_t^2 N_r = \frac{64\pi R^4 k T_{eff} L}{\sigma\lambda^2} \left(\frac{\Omega}{T_f}\right)\left(\frac{E}{N_0}\right) \frac{1}{\omega_t} .$$

Note that for a filled transmit array of half-wave spaced elements,

$$L_{x_t} \cdot L_{y_t} = N_{x_t} \cdot N_{y_t} \cdot \frac{\lambda^2}{4} = N_t \frac{\lambda^2}{4} .$$

Therefore from (76e), the solid angle of the transmitted beam for a filled array of half-wave spaced elements is

$$(78) \qquad \omega_t = \frac{4}{N_t} .$$

Substituting Eq. (78) into Eq. (77) yields exactly the same search parameter as in Eq. (8), and this is as it should be. Since the array cost has been shown to decrease with decreasing Γ_s (e.g., see Eq. (29)), it would be well to try to reduce Γ_s in Eq. (77) by choosing a transmit beamwidth that is greater than that produced by the filled array (i.e., Eq. (78)). However, this requires the use of average element spacings of less than one-half wavelength; and since such spacings are not practicable, we see that thinning of the transmitting array is not a fruitful approach to cost savings for search radar arrays. The receive array can be thinned without search performance degradation if a bundle of receive beams is formed to match the transmitter beam. For fixed performance, no cost saving results since receive area (on each of the multiple receive beams) is proportional to the total number of elements that cannot be reduced without degradation. In fact, the need to form multiple receive beams will in general result in added complexity and cost. If the same array is to also be used for tracking, then thinning the receive array would be desirable since it results in reduced cost for fixed tracking performance, as we shall now see.

TRACKING

For tracking with a thinned receive array the angle accuracies of Eq. (10) become

(79)
$$\sigma_{\theta_x} = \frac{\sqrt{3}\lambda}{\pi L_{x_r}\sqrt{\frac{2E}{N_0}}} = \frac{\sqrt{3}}{\pi \mathcal{L}_{x_r}\sqrt{\frac{2E}{N_0}}} ,$$

$$\sigma_{\theta_y} = \frac{\sqrt{3}\lambda}{\pi L_{y_r}\sqrt{\frac{2E}{N_0}}} = \frac{\sqrt{3}}{\pi \mathcal{L}_{y_r}\sqrt{\frac{2E}{N_0}}} ,$$

from which

(80)
$$\frac{E}{N_0} = \frac{3}{2\pi^2 \sigma_{\theta_x} \sigma_{\theta_y} \mathcal{L}_{x_r} \mathcal{L}_{y_r}} .$$

Substituting Eqs. (76a, b, c) into Eq. (2) yields as an alternate expression for E/N_0:

(81)
$$\frac{E}{N_0} = \bar{P}_t N_t^2 N_r \frac{\sigma \lambda^2 T_d}{64\pi R^4 k T_{eff} L} .$$

Equating Eq. (80) and Eq. (81) yields as the tracking parameter

(82)
$$\Gamma_t = \bar{P}_t N_t^2 N_r = \frac{96 R^4 k T_{eff} L}{\pi \sigma \lambda^2 (\mathcal{L}_{x_r} \mathcal{L}_{y_r}) \sigma_{\theta_x} \sigma_{\theta_y}} \cdot \frac{1}{T_d} .$$

It will add to the clarity of the discussion that follows to introduce at this time, in place of the parameter $\mathcal{L}_{x_r} \cdot \mathcal{L}_{y_r}$ the corresponding number of elements N'_{r_F} that the thinned receive array would contain if it were filled with half-wave spaced elements. Thus,

(83)
$$N'_{r_F} = \frac{L_x}{D_x} \frac{L_y}{D_y} = \frac{4 L_x L_y}{\lambda^2} = 4 \mathcal{L}_x \mathcal{L}_y = \frac{4 A_r}{\lambda^2} ,$$

where A_r is the physical area of the receive array.

In the following analysis primed superscripts will be used to denote parameters associated with the thinned array design. Unprimed quantities will refer to parameters associated with the design of arrays of uniformly spaced elements.

Thus Eq. (82) becomes

(84)
$$\Gamma'_t = \bar{P}_t N'^2_t N'_r = \left(\frac{384 R^4 k T_{eff} L}{\pi \sigma \lambda^2 \sigma_{\theta_x} \sigma_{\theta_y} T_d} \right) \frac{1}{N'_{r_F}} .$$

Since the cost of a tracking radar decreases with decreasing Γ_t (e.g., see Eq. (37)), it follows from Eqs. (83) and (84) that the radar cost can be monotonically decreased by increasing the physical receive array area, or equivalently, by increasing the parameter N'_{r_F}. This result follows from the implicit assumption in the cost equation (21) that the array cost depends on the number of transmitter and receiver elements and not on the array area. The above statements are more clearly exhibited by the introduction of a new parameter that we will call the thinness ratio, T_R, given by

(85) $$T_R = \frac{N'_r}{N'_{r_F}} .$$

As the name indicates, T_R is equal to the ratio of the number of receiver elements used to the number required to fill the receive array. Substituting Eq. (85) into (84) yields

(86) $$\Gamma'_t = \bar{p}_t N'^2_t N'^2_r = T_R \left(\frac{384 R^4 k T_{eff} L}{\pi \sigma \lambda^2 \sigma_{\theta_x} \sigma_{\theta_y} T_d} \right) .$$

Comparing Eq. (86) with Eq. (20), we see that

(87) $$\Gamma'_t = T_R \Gamma_t .$$

It follows that if the cost formula of Eq. (21) is used, the optimum values of N'_t, N'_r, and $C'_{\substack{opt\\track}}$ will be given by Eqs. (36a, b) and (37), where Γ_t of Eq. (20) is replaced by $T_R \Gamma_t$ from Eq. (87). Thus

(88a) $$N'_{t_{opt}} = T_R^{1/4} \left[\frac{\Gamma_t}{\bar{p}_t} \frac{C_r^2}{(C_t + C_p \bar{p}_t)^2} \right]^{1/4} ,$$

(88b) $$N'_{r_{opt}} = T_R^{1/4} \left[\frac{\Gamma_t}{\bar{p}_t} \frac{(C_t + C_p \bar{p}_t)^2}{C_r^2} \right]^{1/4} ,$$

and

$$C'_{\substack{\text{opt}\\\text{track}\\\text{(thinned)}}} = 2T_R^{\frac{1}{2}}\left(\frac{\Gamma_t}{\bar{p}_t}\right)^{\frac{1}{2}}\left[C_r(C_t + C_p\bar{p}_t)\right]^{\frac{1}{2}} . \tag{89}$$

Comparing Eq. (89) and Eq. (37), we obtain the simple relationship shown in Eq. (90):

$$\frac{C'_{\substack{\text{opt}\\\text{track}\\\text{(thinned)}}}}{C_{\substack{\text{opt}\\\text{track}\\\text{(filled)}}}} = T_R^{\frac{1}{2}} . \tag{90}$$

This result is plotted in Fig. (10).

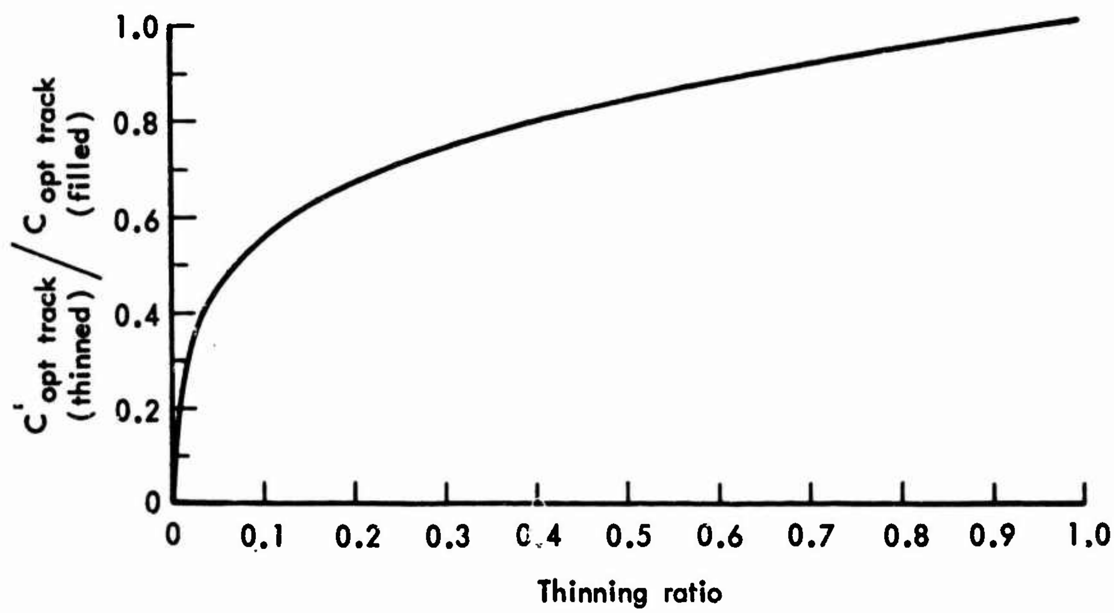

Fig. 10--Ratio of the thinned to unthinned tracking array cost as a function of the thinness ratio

In practice, the idealized cost improvement predicted by Fig. 10 cannot be achieved, particularly for very thin arrays where there will certainly be a cost penalty associated with extremely wide element spacing. Even in the absence of a significant structure cost, the cost of the feed lines between the elements would eventually be comparable to the module cost.

A more realistic cost equation for thinned arrays should include a cost term proportional to receive array area. Modifying Eq. (21) to include such a term, we have

$$C' = C_t N'_t + C_p \bar{p}_i N'_t + C_r N'_r + C_A A_r \,, \tag{91}$$

where C_A is the cost per unit area for the receive array and A_r is the physical area of the thinned receive array. Using Eqs. (83) and (85), we have

$$A_r = \frac{\lambda^2}{4} N'_{r_F} = \frac{\lambda^2}{4} \frac{N'_r}{T_R} \,. \tag{92}$$

Substituting Eq. (92) into (91), we have

$$C' = (C_t + C_p \bar{p}_t) N'_t + \left(C_r + \frac{C_A \lambda^2}{4T_R} \right) N'_r \,. \tag{93}$$

This equation may now be optimized for minimum cost using a Lagrange multiplier subject to the constraint given by the thinned tracking parameter Γ'_t of Eq. (86). Equation (93) is identical in form to Eq. (21) except that C_r in Eq. (21) is replaced by

$$\left(C_r + \frac{C_A\lambda^2}{4T_R}\right)$$

in Eq. (93). Once again by analogy with Eqs. (31) through (37), we can write for the optimum values of N_t', N_r', and $C'_{\substack{opt\\track}}$ as follows:

(94a) $$N'_{t\,opt} = T_R^{\frac{1}{4}}\left(\frac{\Gamma_t}{\bar{P}_t}\right)^{\frac{1}{4}}\left[\frac{C_r + \frac{C_A\lambda^2}{4T_R}}{C_t + C_p\bar{P}_t}\right]^{\frac{1}{2}},$$

(94b) $$N'_{r\,opt} = T_R^{\frac{1}{4}}\left(\frac{\Gamma_t}{\bar{P}_t}\right)^{\frac{1}{4}}\left[\frac{C_t + C_p\bar{P}_t}{C_r + C_A\lambda^2/4T_R}\right]^{\frac{1}{2}}.$$

Substituting this expression for N_r' into Eq. (92), we obtain for the area occupied by the thinned receive array

(94c) $$A_r = \frac{\lambda^2}{4T_R^{\frac{3}{4}}}\left(\frac{\Gamma_t}{\bar{P}_t}\right)^{\frac{1}{4}}\left[\frac{C_t + C_p\bar{P}_t}{C_r + C_A\lambda^2/4T_R}\right]^{\frac{1}{2}},$$

where Γ_t is still given by Eq. (20).

Finally,

(95) $$C'_{\substack{opt\\track\\(thinned)}} = 2T_R^{\frac{1}{4}}\left(\frac{\Gamma_t}{\bar{P}_t}\right)^{\frac{1}{4}}\cdot\left[\left(C_r + \frac{C_A\lambda^2}{4T_R}\right)\left(C_t + C_p\bar{P}_t\right)\right]^{\frac{1}{2}}.$$

For a given thinness ratio, T_R, Eq. (95) is the minimum cost for a tracking radar having a specified tracking parameter Γ_t and fixed coefficients C_t, $\bar{P}_t$, C_r, C_p, and C_A. Examination of the two product terms containing T_R in Eq. (95) shows that as T_R is decreased, one term increases while the second decreases, indicating the possible existence of an optimum choice for T_R to minimize cost. From Eq. (95) it is clear that the only parameters that determine whether or not a cost minimum exists versus T_R are C_r, C_A, and λ.

Taking the derivative of Eq. (95) with respect to T_R and equating to zero yields the following equation for the optimum thinness ratio:

(96) $$T_{R_{min}} = \frac{C_A \lambda^2}{4C_r} .$$

Depending upon the specific values for C_A, C_r, and λ, either of two possible conditions may exist. Either $C_A\lambda^2/4C_r < 1$, in which case it is profitable to thin, or $C_A\lambda^2/4C_r > 1$, in which case it is not profitable (i.e., $T_{R_{min}} > 1$ results in a greater cost in agreement with Eq. (90)).

To exhibit these results Eq. (95) was evaluated using fixed values of $\Gamma_t = 10^{14}$, C_t = \$1100, and C_p = \$5/watt, together with several different values for C_r, C_A, and λ as indicated below:

C_r = \$100, \$1000;

C_A = 25, 50, 75, and 100 $\$/ft^2$;

λ = 2.24 ft (UHF), 0.852 ft (L band), and 0.1 ft (X band).

In Fig. 11, several representative curves of $C'_{\substack{opt\\ track\\ (thinned)}}$ versus T_R are plotted. In these curves the thinness ratio was limited to values greater than 0.05 since it was felt that smaller ratios would not be practicable. The curves of Fig. 11 illustrate the various possible results of thinning the receive array. These results may be interpreted as follows:

Curve 1 shows that for fixed λ, if the area cost coefficient, C_A, is sufficiently large relative to the cost of a receive module, C_r, then thinning is not profitable. For the cost coefficients in curve 1, Eq. (96) predicts $T_{R_{min}} > 1$ so that the thinned array would actually be more costly than an unthinned array ($T_R = 1$). Curve 2 shows that if C_r is sufficiently large with respect to C_A, then the optimum thinness ratio will approach zero, as predicted by Eq. (96). Actually for the case of curve 2, a minimum value of T_R is predicted at $T_{R_{min}} = 0.03$ but is not shown since it is less than the arbitrary minimum value of $T_R = 0.05$ noted earlier. Curves 3 and 4 illustrate that between the two extreme conditions exhibited by curves 1 and 2, there is a range of ratios of C_r to C_A such that useful optimum thinness ratios do exist (i.e., $0.05 < T_{R_{min}} < 1$). Curves 3 and 4 further illustrate that, as predicted by Eq. (96), when a cost minimum does exist, the larger the ratio of C_r to C_A, the lower the optimum thinness ratio.

The results in Fig. 11 are all for $\lambda = 2.24$ ft (UHF). The results for the smaller wavelengths $\lambda = 0.852$ ft (L band) and $\lambda = 0.1$ ft (X band) were h that for all combinations of C_r and C_A, thinning always produced a cost reduction (i.e., there were no cases similar to

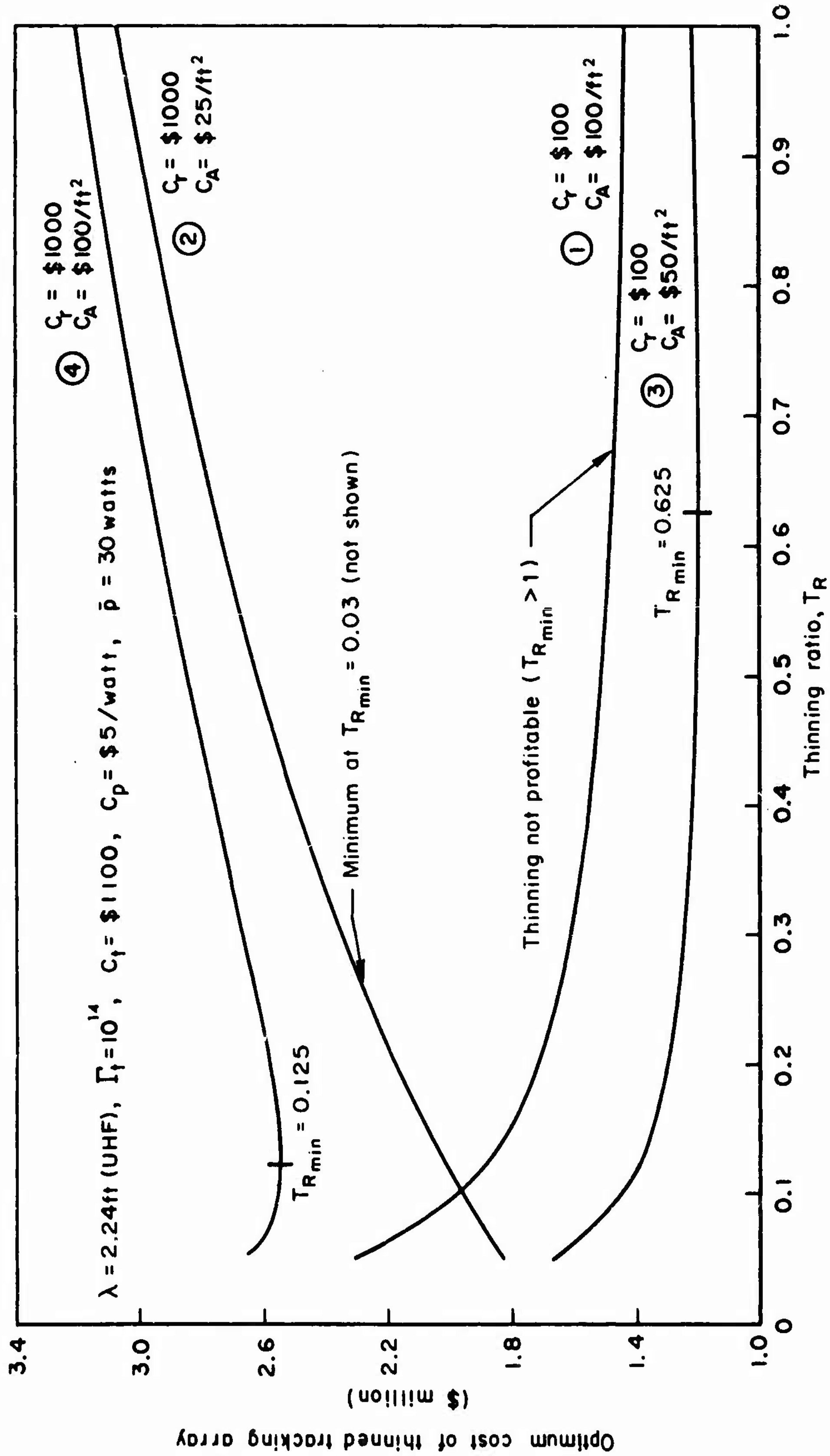

Fig. 11--Optimum cost of thinned tracking array radar versus thinning ratio

curve 1). At X band, the results were such that for all cases the results were of the form of curve 2 so that it would be desirable to thin down to $T_R = 0.05$ barring any other size constraints.

In conclusion, it has been shown that thinning of the receive array can reduce the cost of tracking radars, and for radars combining search and track functions, the thinning of the receive array can be applied without degrading search if multiple receive beams are used.

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10. ABSTRACT

Cost estimating relationships between the performance requirements of phased array radars and their hardware and installation cost are developed for both the search and tracking functions. Appropriate performance parameters for each function are derived from system operational requirements, and Lagrange multipliers are used to obtain the optimum design formulas and minimum-cost equations. Results of several phased array cost examples are analyzed to test the sensitivity of the cost estimates to variations in required system performance and in the cost coefficients of the cost equations. The use of higher power modules at lower cost per watt results in less expensive search arrays. For tracking array designs, the situation is reversed. By a compromise design, the same array can be used for both functions.

11. KEY WORDS

Radar
Cost estimating relationships
Cost analysis